M 11410

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Regular) Examination, November 2011

CORE COURSE IN MATHEMATICS

5B08 MAT : Graph Theory

Time: 3 Hours Max. Weightage : 30

***Instruction** : Answer to all questions.*

1. Fill in the blanks :

   a) The 3-cube Q_3 has ____________ vertices.

   b) The complete graph K_n has ____________ different spanning trees.

   c) The complete graph K_3 has ____________ different Hamiltonian cycles.

   d) Suppose G is a 2-regular graph of 5 vertices. Then the order of incidence matrix of G is ____________. **(Wt. 1)**

Answer **any six** from the following **(Wt. 1 each)** :

2. Define a graph isomorphism.

3. Define the join of two graphs G_1 and G_2 with no vertex in common.

4. Define Euler and Hamiltonian graphs.

5. Give an example of a matching in G which is maximum but not perfect.

6. Draw the strongly connected orientations of K_3.

7. Draw the (3, 2) de Brujin diagram and use it to construct a (3, 2) Brujin sequence.

8. When a graph G is said to be unicyclic ? Give example.

9. Define articulation point of a graph G.

10. Draw a graph which has a Hamiltonian path but no Hamiltonian cycle.

P.T.O.

Answer **any seven** of the following (Wt. **2 each**) :

11. Prove that in any graph G there is an odd number of odd vertices.
12. Define the square of a simple connected graph G. Show that the square of $K_{1,3}$ is K_n.
13. Let G be a graph with n vertices and let A denote the adjacency matrix of G. Let $B = (b_{ij})$ be the matrix $B = A + A^2 + ... + A^{n-1}$. Prove that G is connected iff B has no zero entries off the main diagonal.
14. Prove that if T is a tree with n vertices then it has precisely (n – 1) edges.
15. Prove that a graph G is connected if and only if it has a spanning tree.
16. Let G be a simple connected graph with at least two vertices and let v be a vertex in G of smallest possible degree say K then prove that $K(G) \leq K$.
17. Prove that a simple graph G is Hamiltonian if and only if its closure C (G) is Hamiltonian.
18. Prove that a connected graph G has an Euler trail if and only if it has at most two odd vertices.
19. Prove that every tournament T has a directed Hamiltonian path.
20. Prove that for each pair of positive integer n and k, both greater than one, the de Brujin diagram $D_{n,k}$ has a directed Euler tour.

Answer **any three** of the following (Wt. **3 each**) :

21. Let G be a non-empty graph with atleast two vertices. Then prove that G is bipartite if and only if it has no odd cycles.
22. Let e be an edge of a graph G and as usual let G – e be the subgraph obtained by deleting e. Then prove that $W(G) \leq W(G-e) \leq W(G) + 1$.
23. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.
24. If G is a simple graph with n vertices where $n \geq 3$ and the degree $d(v) \geq n/2$ for every vertex v of G, then prove that G is Hamiltonian.
25. Prove that a strongly connected tournament T on n vertices contains directed cycles of length 3, 4, ... n.
