M 11411

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Regular) Examination, November 2011

CORE COURSE IN MATHEMATICS

5B09 MAT : Differential Equations and Numerical Analysis

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks (**Weightage : 1**) :

   a) If $y_1(x)$ and $y_2(x)$ are linearly independent solutions of the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$. Then its general solution is ________

   b) Let m_1 and m_2 be the roots of the characteristic equation of $y'' + py' + qy = 0$. If m_1 and m_2 are real and distinct, then the general solution is ________

   c) The characteristic equation of $y'' + 2y' + y = 0$ is ________

   d) Consider the non homogeneous equation $y'' + py' + qy = R(x)$, where $R(x) = e^{ax}$. Then if a is a single root of the auxiliary equation, the particular solution is ________

Answer **any six** from the following (**Weightage 1 each**) :

2. Solve $(y - x^3)\, dx + (x + y^3)\, dy = 0$.

3. State the existence and uniqueness theorem.

4. If k and b are positive constants, find the general solution of $y'' + k^2 y = \sin bx$.

5. Show that e^x and e^{-x} are linearly independent solutions of $y'' - y = 0$ on any interval.

6. State the empirical laws taken as the basis of investigation of one dimensional heat flow.

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7. What are the initial and boundary conditions of wave equation ?

8. Evaluate $\sqrt{28}$ to four decimal places by Newton's iterative method.

9. Explain the Euler's method for solving the first order differential equation.

10. Using Picard's method, find a solution of $\frac{dy}{dx} = 1 + xy$ upto the third approximation, when y (0) = 0.

Answer **any seven** from the following **(Weightage 2 each)** :

11. Solve $y(xy + 2x^2y^2)\ dx + x(xy - x^2y^2)\ dy = 0$.

12. Solve the differential equation $x^4 \frac{dy}{dx} + x^3 y = -\sec(xy)$.

13. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$.

14. Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y'' + y = 0$ on any interval and find the particular solution for which y (0) = 2 and y′(0) = 3.

15. Find the general solution of $y'' + 10y' + 25y = 14e^{-5x}$.

16. Using the method of separation of variables, solve the equation $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$.

17. If a string of length l is initially at rest in the equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = V_0 \sin^3 \frac{\pi x}{l}, 0 < x < l$, determine the displacement y (x, t).

18. Given sin 45° = 0.7071, sin 50° = 0.7660, sin 55° = 0.8192, sin 60° = 0.8660 find sin 52°, using Newton's interpolation formula.

19. Perform two iterations of Picard's method to find an approximate solution of the initial value problem $\frac{dy}{dx} = x + y^2$; y (0) = 1.

Answer **any three** from the following **(Weightage 3 each)** :

20. Use Gauss -Jordan reduction method to compute the inverse of the matrix $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by applying elementary row transformations.

21. Solve $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.

22. Solve $3e^x \tan y\,dx + (1 + e^x)\sec^2 y\,dy = 0$, given $y = \left(\frac{\pi}{4}\right)$ when $x = 0$.

23. Solve by the method of variation of parameters $y'' + 2y' + y = e^{-x}\log x$.

24. Using Euler's method, solve for y at x = 0.1 from $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$ taking step size h = 0.025.

25. Use Taylor's method to obtain approximate value of y at x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.

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