M 11409

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Regular) Examination, November 2011
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra

Time: 3 Hours Max. Weightage : 30

1. Mark **each** of the following are **true** or **false**. **(Weightage : 1)**
   a) In a group, each linear equation has a solution
   b) Any two groups of three elements are isomorphic
   c) Every abelian group is cyclic
   d) In every cyclic group, every element is a generator.

Answer **any 6** from the following **9** questions (Weightage **1 each**)

2. Define a group. Let * be defined on Z by letting a*b = ab. Is Z a group under the operation*. Justify your answer.

3. Show that every group G with identify e and such that $x * x = e$ for all $x \in G$ is abelian.

4. Express the following permutation of {1, 2, 3, 4, 5, 6, 7, 8} as a product of disjoint cycles, and then as a product of transpositions.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$

5. Let H be a subgroup of a group G and let $a \in G$. Define the left and right cosets of H containing a. Exhibit all left and right cosets of the subgroup $\langle 4 \rangle$ of Z_{12}.

P.T.O.

6. Define a group homomorphism. Determine whether the map $\phi : R \to Z$ under addition given by $\phi(x)$ = the greatest integer $\leq x$ is a homomorphism.

7. Determine the number of group homomorphisms from Z onto Z.

8. Define a ring. Give an example of a ring with unity $1 \neq 0$ than has a subring with non-zero unity $1' \neq 1$.

9. Define the characteristic of a ring R. Find the characteristic of the ring $Z_3 \times Z_3$.

10. Show that every field F is an integral domain.

Answer **any 7** from the following **10** questions. (Weightage **2 each**)

11. Show that a group with no proper nontrivial subgroups is cyclic.

12. Discuss the different group structures of order 4.

13. Show that if G is a finite group with identity e and with an even number of elements, then there is $a \neq e$ in G such that $a*a=e$.

14. Show that a group with atleast two elements but with no proper nontrivial subgroups must be finite and of prime order.

15. Obtain the group of symmetries of the square with vertices 1, 2, 3 and 4. Show that it is nonabelian.

16. Define kernel of a homomorphism. Show that a group homomorphism $\phi : G \to G'$ is a one-to-one map if and only if $Ker(\phi) = \{e\}$.

17. Define an automorphism of a group G. Show that all automorphisms of a group G form a group under function composition.

18. Show that if a finite group G contains a nontrivial subgroup of index 2 in G, the G is not simple.

19. Show that every finite integral domain is a field.

20. Find all solutions of the congruence $12x \equiv 27 \pmod{18}$.

Answer **any 3** from the following **5** questions. (Weightage **3 each**)

21. Let G be a cyclic group with generator a. If the order of G is infinite, then show that G is isomorphic to $\langle Z, + \rangle$. If G has finite order n, then show that G is isomorphic to $\langle Z_n, + \rangle$.

22. State and prove Cayley's theorem.

23. State and prove Lagrange's theorem.

24. Let H be a normal subgroup of G. Then show that the cosets of H form a group G/H under the binary operation (aH) (bH) = (ab) H.

25. If a is an integer relatively prime to n, then show that $a^{\phi(n)} - 1$ is divisible by n.