M 8563

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, May 2015
CORE COURSE IN MATHEMATICS
4B04 MAT : Calculus

Time : 3 Hours Max. Weightage : 30

Fill in the blanks :

1. a) ______________ is an example of a function which is continuous at x = 0 and has no derivative at x = 0.

   b) $\frac{d}{dx}\left(1-x^2\right)^{-1/2} =$ ______________

   c) If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$, then $\lim_{x \to 0} f(x) =$ ______________

   d) The function $y = \sin\left(\frac{1}{x}\right)$ has no limit as $x \to$ ______________ **(Weight : 1)**

2. a) $\int \frac{2z}{\sqrt[3]{z^2+1}}\,dz =$ ______________

   b) $\int_{1}^{1} 5x^4 \sqrt{x^5+1}\,dx =$ ______________

   c) $\Gamma(n) =$ ______________

   d) $\int_{0}^{\infty} e^{-x^2}\,dx =$ ______________ **(Weight : 1)**

Answer **any five** from the following **(Weight 1 each) :**

3. Find :

   a) $\lim_{x \to 0} \frac{\sin x - x}{x^3}$

   b) $\lim_{\theta \to 0} \left(\frac{1}{\theta} - \frac{1}{\sin\theta}\right)$.

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4. State the maximum-minimum theorem for continuous functions.
5. Find two positive numbers whose sum is 20 and whose product is as large as possible.
6. State the Mean-Value Theorem.
7. Show that the equation $x^3 + 3x + 1 = 0$ has exactly one root.
8. State Rolle's theorem.
9. Replace $(x - 5)^2 + y^2 = 25$ by a polar equation.
10. Find b for which $f(x) = x^3 + bx^2 + cx + d$ has a point of inflexion at $x = 1$; where a, b, c, d are constants. **(5x1=5)**

Write **any seven** from the following **(Weight 2 each)** :

11. Find n^{th} derivatives of :

 a) $e^x \cos 2x$

 b) $\frac{x+1}{x^2-4}$

12. State Leibnitz's theorem and use it to prove that if $y = e^{a \sin^{-1} x}$,

 $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 + a^2)\, y_n = 0.$

13. Prove that the asymptotes of $x^2y^2 = c^2 (x^2 + y^2)$ are the sides of a square.
14. Using Maclaurin's series, obtain the expansion of $e^x \sin x$ up to the term containing x^5.
15. Find the radius of curvature at (x, y) for the curve $a^2y = x^3 - a^3$.
16. Find the evolute of the parabola $y^2 = 4ax$.
17. Evaluate :

 a) $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$

 b) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$.

18. Find the length of the curve y = long sec x between the points given by x = 0 and $x = \frac{\pi}{3}$.

19. One arc of the sine curve y = sin x revolves round the x-axis. Find the volume of the solid so generated.

20. Find the area enclosed by the Cardioid $r = a(1 + \cos\theta)$. **(7×2=14)**

Write **any three** from the following **(Weight 3 each)** :

21. Find $\frac{dy}{dx}$ for the following :

    a) If $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$

    b) If $x^y = y^x$, prove that $\frac{dy}{dx} = \frac{y(y - x\log y)}{x(x - y\log x)}$.

    c) If $y = (1+\log x)^{x^x}$.

22. State and prove the fundamental theorem of Calculus.

23. Use Simpson's rule with h = 4 to evaluate $\int_0^1 5x^4 dx$.

24. Use reduction formula to evaluate :

    a) $\int x^n e^{ax} dx$

    b) $\int x^n \sin x \, dx$.

25. a) Find the perimeter of the Cardioid $r = a(1 - \cos\theta)$.

    b) Find the volume of the solid obtained by revolving the Cardioid $r = a(1 + \cos\theta)$ about the initial line. **(3×3=9)**
