M 6328

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CCSS – Regular/Supple./Improv.) Examination, May 2014
CORE COURSE IN MATHEMATICS
4B04 MAT : Calculus

Time: 3 Hours Max. Weightage : 30

Fill in the blanks :

1. a) The function $y = |x|$ is continuous at $x = 0$ but not ________

   b) The function $y = \frac{\sin x}{x}$ has a ___________ discontinuity at $x = 0$.

   c) If $\sqrt{5 - 2x^2} \le f(x) \le \sqrt{5 - x^2}$, $\lim_{x \to 0} f(x) =$ _____________

   d) The function $y = \sin\left(\frac{1}{x}\right)$ has no limit as $x \to$ _______ **(W=1×1=1)**

2. a) $\int_0^2 \frac{6x^2}{\sqrt{2x^3 + 9}}\, dx =$ _______________

   b) State the integral existence theorem.

   c) Evaluate $\int \frac{(z+1)}{\sqrt[3]{3z^2 + 6z + 5}}\, dz$.

   d) $\Gamma(10) =$ _____________ **(W=1×1=1)**

P.T.O.

Answer **any 5** from the following (Wt. **1 each**) :

3. a) Find $\lim_{x \to \infty}\left(2 + \frac{\sin x}{x}\right)$.

   b) $\lim_{\theta \to 0}\left(\frac{1}{\theta} - \frac{1}{\sin\theta}\right)$.

4. State maximum-minimum theorem for continuous function.

5. Find two positive numbers whose sum is 20 and whose product is as large as possible.

6. State Mean value theorem.

7. Show that equation $x^3 + 3x + 1 = 0$ has exactly one root.

8. Find the absolute maximum and minimum values of the function $y = x^3 - 3x + 2$, $0 \le x \le 2$ on the closed interval [0, 2].

9. Replace $(x - 2)^2 + y^2 = 4$ by a polar equation.

10. Find b for which $f(x) = x^3 + bx^2 + cx + d$ have a point of inflexion at x = 1; where a, b, c, d are constants. **(5×1=5)**

Write **any seven** from the following (Wt. **2 each**) :

11. Find the n^{th} derivative of $e^{4x}\cos 3x$.

12. State Leibuitz' theorem and use it to prove that if $y = e^{\tan^{-1}x}$

    $(1 + x^2)y_{x+2} + [2(n + 1)x - 1]y_{n+1} + x(n + 1)y_n = 0$.

13. Find the asymptotes of $y^3 + x^2y + 2xy^2 - y + 1 = 0$.

14. Find the Maclaurin's series expansion of y = log cos hx.

15. Find the radius of curvature at any 'θ' on the curve $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$.

16. Find the evolute of four Cusped hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$.

17. Show that :

i) $\int_0^{\infty} \frac{dx}{1+x^2}$ is convergent.

ii) $\int_1^{\infty} \frac{dx}{x}$ is divergent.

18. Find the area enclosed by the cardioid. $\gamma = a(1+\cos\theta)$

19. One arch of the sine curve y = sinx revolves round the x-axis. Find the volume of solid so generated.

20. Find the length of the curve y = log sec x between the points given by x = 0 and $x = \pi/3$. **(7×2=14)**

Write **any 3** from the following (Wt. **3 each**) :

21. Find $\frac{dy}{dx}$

a) $y = \sin^m x \cdot \cos^4 x \cdot \cosh^2 x$.

b) if $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

c) $x = a(\theta + \sin\theta); y = a(1-\cos\theta)$.

d) $y = \tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$.

22. State and prove fundamental theorem of Calculus.

23. Use Simpson's rule with x = 4 to evaluate $\int_0^1 5x^4 dx$.

24. Use reduction formula to evaluate :

a) $\int_0^{\pi/2} \sin^5 x \cos^3 x dx$

b) $\int x^m (\log x)^n$ when m and n are integers.

25. a) Find the area of the region enclosed by the parabola $y^2 = 4ax$ and $x^2 = 4by$.

b) Find the volume of the solid obtained by revolving the Cardioid $\gamma = a(1 + \cos\theta)$ about the initial line. **(3×3=9)**
