



M 3138

Reg. No. :

Name :

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./
B.A. Afsal-UI-Ulama Degree (CCSS – Reg./Supple./Improv.)

Examination, May 2013

CORE COURSE IN MATHEMATICS

6B13 MAT : Integral Transforms

Time: 3 Hours

Max. Weightage: 30

1. Fill in the blanks :

a) Laplace transform of e^{at} is _____

b) If $m \neq n$, the value of $\int_{-\pi}^{\pi} \cos mx \cos nx \, dx =$ _____

c) A function $f(x)$ is said to be odd if $f(-x) =$ _____

d) $Z(1) =$ _____ (Weightage 1)

Answer **any six** from the following (Weightage **1 each**) :

2. State Convolution theorem for Laplace transform.

3. Find $L(t \cos at)$.

4. Find the inverse Laplace transform of $\frac{s^2 + 6}{(s^2 + 1)(s^2 + 4)}$.

5. Explain the complex form of Fourier series expansion.

6. Write the relation between Unit sample sequence and Unit step sequence.

7. Find Z-transform of $\sin n\theta$.

8. Find $Z^{-1} \left\{ \frac{3}{(3z - 1)} \right\}$.

P.T.O.



9. Explain Fourier sine and cosine transforms of functions.
10. Find Fourier cosine transform of

$$f(x) = \begin{cases} k & 0 < x < a \\ 0 & x > a \end{cases}$$

(Weightage 6×1=6)

Answer **any seven** from the following (Weightage **2 each**).

11. Define Dirac's delta function. Also find its Laplace transform.
12. Convert into unit step function and hence find the Laplace transform of

$$f(t) = \begin{cases} 2, & 0 < t < \pi \\ 0 & \pi < t < 2\pi \\ \sin t & t > 2\pi \end{cases}$$

13. Find the Fourier series expansion of e^x in the interval $0 < x < 2\pi$.
14. Express $f(x) = x$ as a half range sine series in $0 < x < 2$.
15. Find the complex Fourier series of $f(x) = e^{ax}$, $-\pi < x < \pi$.
16. State and prove final value theorem for the Z-transform.
17. Find the Z-transform of $f * g$ where $f(n) = 3^n u(n)$ and $g(n) = 4^n u(n)$.

18. Using partial fraction method, find the inverse Z-transform of $\frac{z^3 - 20z}{(z - 2)^3 (z - 4)}$.

19. Prove that $\int_0^{\infty} \frac{\cos xw}{1 + w^2} dw = \frac{\pi}{2} e^{-x}$, $x > 0$.

20. With usual notation, prove that

$$F_s \{f''(x)\} = -w^2 F_s \{f(x)\} + \sqrt{\frac{2}{\pi}} w f(0).$$

(Weightage 7×2=14)



Answer **any three** from the following (Weightage **3 each**).

21. Solve the integral equation

$$y(t) = t + \int_0^t y(T) \sin(t - T) dT.$$

22. Using Laplace transform, solve the initial value problem :

$$y'' + 2y' + 2y = r(t); y(0) = 1, y'(0) = -5 \text{ where}$$

$$r(t) = \begin{cases} 10 \sin 2t, & \text{if } 0 < t < \pi \\ 0 & \text{if } t > \pi \end{cases}$$

23. Obtain the Fourier series for the function $f(x) = \begin{cases} l - x & 0 \leq x \leq l \\ 0, & l \leq x \leq 2l \end{cases}$

$$\text{Deduce that } \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$

24. a) Using power series method, find the inverse Z-transform of $\log \left(\frac{z}{z+1} \right)$.

b) Using long division method, find the inverse Z-transform of $\frac{2(z^3 - z)}{(z^2 + 1)^2}$.

25. State and prove Convolution theorem for Fourier transform. **(Weightage 3×3=9)**
