M 3527

Reg. No. : .....

Name : .....

IV Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal-Ul-Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, May 2013

CORE COURSE IN MATHEMATICS

4B04 MAT : Calculus

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) _____ is an example of a function which is not continuous.

   b) The $(n-1)^{th}$ derivative of e^{bx} is _____

   c) $\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}$ is _____

   d) _____ is an example of a function which is not differentiable in $(-2, 3)$. **(W=1)**

2. a) $\int 2x dx =$ _____

   b) $\int x^{\frac{1}{3}} dx =$ _____

   c) $\int \sin(2x+3)\, dx =$ _____

   d) $\sum_{k=1}^{n} k^3 =$ _____ **(W=1)**

Write **any five** from the following :

3. $f(x) = \sqrt{x}$, $L = \frac{1}{2}$, $x_0 = \frac{1}{4}$, $\varepsilon = 0.1$. Find an open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds.

P.T.O.

4. Define left-hand limit.

5. Define tangent curve at a point P.

6. Give an example of a function which is continuous but not differentiable and explain it.

7. If $y = \sqrt{\frac{t}{t+1}}$, find $\frac{dy}{dt}$ using logarithmic differentiation.

8. Evaluate the integral $\int_{\log 4}^{\log 9} e^{t/2} dt$.

9. Solve the initial value problem :

$\frac{dy}{dt} = e^{-t} \sec^2(\pi e^{-t})$, $y(\log 4) = 2/\pi$.

10. Evaluate $\lim_{x \to 0} \frac{x^{2^x}}{2^x - 1}$. **(5×1=5)**

Write **any seven** from the following (Weightage **2 each**) :

11. Find $\cot\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec^{-\frac{1}{2}}\right)$.

12. Evaluate $\int_{-1}^{0} \frac{6dt}{\sqrt{3 - 2t - t^2}}$.

13. Evaluate $\int \operatorname{sech}^2\left(x - \frac{1}{2}\right) dx$.

14. Find the n^{th} derivative of $\sin 2x \cdot \cos 3x$.

15. If $y = \sin(m \sin^{-1} x)$ prove that :

$(1 - x^2) y_2 - x y_1 + m^2 y = 0$ and

$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0$

16. Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

17. Suppose that f is continuous on [a, b] and f is differentiable on (a, b). If $f' > 0$ at each point of (a, b) then show that f increases on [a, b].

18. Replace the polar equation $r\cos\theta + r\sin\theta = 1$ by equivalent Cartesian equation.

19. Find the radius of curvature at the point θ on the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$.

20. Graph the integrand and use area to evaluate the integral $\int_{-2}^{4}\left(\frac{x}{2}+3\right)dx$. **(7×2=14)**

Write **any three** from the following (weightage **3 each**) :

21. i) State and prove the mean value theorem for Definite integrals.

    ii) At what point or points into given interval does the function $f(x) = x^2 - 1$ on $\left[0, \sqrt{3}\right]$ assume its average value.

22. Use Simpson's rule with $n = 4$ to approximate $\int_{0}^{1} 5x^4 dx$. Also estimate the error.

23. i) Evaluate $\int_{0}^{1} \frac{dx}{\sqrt{x\log\left(1/x\right)}}$.

    ii) Evaluate $\int_{0}^{\infty} \frac{x}{1+x^6}dx$.

24. i) Find the areas of the region enclosed by the curves $y = 2\sin x$ and $y = \sin 2x$, $0 \le x \le \pi$.

    ii) Find the volume of the solids generated by revolving the regions bounded by the curves $y = x^2 + 1$, $y = x + 3$ about the x-axis.

25. Graph the function $y = \frac{1}{2x+4}$. **(3×3=9)**
