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Reg. No. : ....

Name : ....

IV Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. (CCSS – Regular) Degree Examination, March 2011
MATHEMATICS (Core Course)
4B04 MAT – Calculus

Time : 3 Hours   Max. Weightage : 30

1. Fill in the blanks : **(W – 1)**

   a) ________ is an example of a function which is continuous on [0, 1]

   b) The n^{th} derivative of e^{ax} is ________

   c) $\lim\limits_{x \to 0} 4x^2 + 3x + \frac{1}{2}$ is ________

   d) ________ is an example of a function which is not differentiable.

2. a) $\int e^{ax} dx =$ ________ **(W – 1)**

   b) $\int \left(3t^2 + t/_2\right) dt =$ ________

   c) $\int \sin(3x + 5)\, dx =$ ________

   d) $\sum\limits_{k=1}^{n} k =$ ____________

Write **any five** from the following (Weightage **1 each**) :

3. Find :

   a) $\lim\limits_{y \to -5} \frac{y^2}{5 - y}$ and   b) $\lim\limits_{x \to 1} \frac{x - 1}{\sqrt{x + 3} - 2}$.

4. Define right-hand limit.

P.T.O.

5. Mention the points to find the tangents to the curve $y = f(x)$ at (x_0, y_0).

6. Show that if f has a derivative at $x = c$, then f is continuous at $x = c$.

7. If $y = \sqrt{\theta+3}\sin\theta$, find $\frac{dy}{d\theta}$ using logarithmic differentiation.

8. Evaluate the interval $\int 8e^{(x+1)}\,dx$.

9. Solve the initial value problem :

$$\frac{d^2y}{dx^2} = 2e^{-x},\ y(0) = 1 \text{ and } y'(0) = 0.$$

10. Evaluate $\lim_{\theta \to 0} \frac{3^{\sin\theta} - 1}{\theta}$.

Write **any seven** from the following (weightage **2 each**).

11. Find $\sin\left[\sin^{-1}\left(-\tfrac{1}{2}\right) + \cos^{-1}\left(-\tfrac{1}{2}\right)\right]$.

12. Evaluate $\int \frac{dy}{y^2 - 2y + 5}$.

13. Show that if u is a differentiable function of x whose values are greater than 1, then $\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{u^2 - 1}}\frac{dy}{dx}$.

14. Find the n^{th} derivative of $\sin^5 x \cos^4 x$.

15. If $y = \left(x + \sqrt{1 + x^2}\right)^m$ prove that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

16. Find the absolute extreme values of $g(t) = 8t - t^4$ on $[-2, 1]$.

17. Suppose that f is continuous on [a, b] and differentiable on (a, b). If $f' < 0$ at each point of (a, b), then show that f decreases on [a, b].

18. Replace the polar equation $r = 4\csc\theta$ by equivalent Cartesian equation.

19. Find the radius of curvature at 't' on the curve $x = 6t^2 - 3t^4$, $y = 8t^3$.

20. Graph the integrand and use area to evaluate the integral $\int_{1/2}^{3/2} (-2x+4)\,dx$.

Write **any three** from the following (weightage **3 each**).

21. State and prove the fundamental theorem of calculus, part I.

22. Prove that

i) $\beta(m,n) = \beta(m, n+1) + \beta(m+1, n)$

ii) $\Gamma\, 1/2 = \sqrt{\pi}$.

23. Using Simpson's rule with n = 4 to approximate $\int_{-1}^{1} (t^3+1)\,dt$.

24. i) Find the areas of the regions enclosed by the curves $y = \sin\left(\pi x/2\right)$ and $y = x$.

ii) Find the volume of the solids generated by revolving the regions bounded by the curve $y = 2\sqrt{x}$, $y = -2$, $x = 0$ about the x-axis.

25. Graph the function $y = \dfrac{x^3+1}{x}$.