Reg. No. :
Name : $\qquad$

# IV Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal UI Ulama Degree (CCSS - Reg./Supple./Improv.) Examination, April 2012 <br> COMPLEMENTARY COURSE IN STATISTICS FOR MATHS AND COMPUTER SCIENCE CORE 4C04 STA : Statistical Inference 

Time: 3 Hours
Max. Weightage : 30

Instruction: Use of scientific calculator permitted. Statistical tables are permitted.

> PART-A

Answer any 10 questions. Weightage 1 each.

1. Mention any two uses of standard error.
2. A sample of size 20 is taken from a normal population with mean 30 and variance 10. If $S^{2}$ is the variance of the sample, what is the probability distribution of $2 S^{2}$ ?
3. Find the mean of chi-square distribution with $n$ degrees of freedom.
4. Write down the density function of F -distribution with $\left(\mathrm{n}_{1}, \mathrm{n}_{2}\right)$ degrees of freedom.
5. Find the variance of student's $t$-distribution.
6. Define consistent estimator.
7. Define sufficient statistic.
8. Define level of significance and power of test.
9. What is confidence interval for a parameter?
10. Why non-parametric tests are called distribution free tests?
11. What is a contingency table ? Write down the expression for the value of chi-square for testing the independence of attributes in a $2 \times 2$ contingency table.

## PART-B

## Answer any 6 questions. Weightage 2 each.

12. Derive the moment generating function of chi-square distribution. Hence establish the additive property of chi-square distribution.
13. Discuss the inter-relationships among normal, chi-square, student's tand F-distributions.
14. Show that the sample variance is always a biased estimator of the population variance. Hence, find an unbiased estimator of the population variance.
15. Let $X_{1}, X_{2}, \ldots X_{n}$ be a sample of size $n$ from a population density function $f(x)=\left\{\begin{array}{l}\frac{1}{\theta} \text { if } 0<x<\theta \\ 0 \\ \text { otherwise }\end{array}\right.$.
Find a sufficient statistic for $\theta$.
16. Derive a $100(1-\alpha) \%$ confidence interval for the difference of the means of two normal populations, stating the assumptions, if any.
17. Obtain the MLE of the parameter $\theta$ of the population density function $f(x)=\frac{1}{2} e^{-|x-\theta|} ;-\infty<x<\infty$ based on a sample of size $n$.
18. To test $H_{0}: \theta=1$ against $H_{1}: \theta=2$, a sample of size one is taken from a population $f(x)=\left\{\begin{array}{ll}1 / \theta & \text { if } 0<x<\theta \\ 0 & \text { otherwise }\end{array}\right.$.
Find the level of significance and power of the test if $\mathrm{H}_{0}$ is rejected when the sample observation is greater than 1.5.
19. Obtain the most powerful size $\alpha$ test for $H_{0}: \theta=\theta_{0}$ against $H_{1}: \theta=\theta_{1}\left(\theta_{1}>\theta_{0}\right)$ for the population $f(x)=\left\{\begin{array}{ll}\theta e^{-\theta x} ; & x>0 \\ 0 & \text { otherwise }\end{array}\right.$ based on a sample of size $n$.
20. Explain the paired $t$-test.

## PART-C

Answer any two questions. Weightage 4 each.
21. Obtain the method of moments estimators of $a$ and $b$ of the population density function $f(x)=\left\{\begin{array}{ll}\frac{1}{b-a} & \text { if } a<x<b \\ 0 & \text { otherwise }\end{array}\right.$ based on a sample of size $n$.
22. To test $H_{0}: \theta=1$ against $H_{1}: \theta=2$ a random sample $\left(X_{1}, X_{2}\right)$ of size 2 is selected from the population $f(x)=\left\{\begin{array}{ll}\theta x^{\theta-1} ; 0<x<1 \\ 0 & \text { otherise }\end{array}\right.$. Find the level of significance and power of the test if the critical region is given by $X_{1} X_{2} \geq 3 / 4$.
23. A survey of 320 families with five children each gave the following distribution :

No. of boys : | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$\begin{array}{lllllll}\text { No. of families : } & 12 & 40 & 88 & 110 & 56 & 14\end{array}$
Use chi-square test to test whether male and female births are equally probable (choose $\alpha=0.05$ ).
24. The following data give the number of hours of service rendered by spark plugs manufactured by two sources.

| Source A : | 200 | 210 | 190 | 200 | 190 | 200 | 180 | 200 | 200 | 210 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Source B : | 190 | 200 | 210 | 190 | 180 | 190 | 200 | 192 |  |  |

Test whether there is any significant difference in average length of service. (choose $\alpha=0.05$ ).

