M 769

Reg. No. : ....

Name : ....

IV Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, April 2012
CORE COURSE IN MATHEMATICS
4B04 MAT : Calculus

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　　Max. Weightage : 30

Fill in the blanks :

1. a) _____ is an example of a function is not continuous at 0.

   b) The $(n-1)^{th}$ derivative of sin x is _____

   c) $\lim\limits_{x \to 0} (x^2 + 3x + 2)$ is _____

   d) _____ is an example of a function which is differentiable in $(-1, 1)$. 　**(W = 1)**

2. a) $\int x^3 \, dx = $ _____

   b) $\int x^{-\frac{1}{3}} \, dx = $ _____

   c) $\int \cos(4x + b) \, dx = $ _____

   d) $\sum\limits_{k=1}^{n} k^2 = $ _____ 　**(W = 1)**

Answer **any five** from the following : 　　　　　　　　　　　　**(Weightage 1 each)**

3. $f(x) = \sqrt{x+1}$, $L = 1$, $x_0 = 0$, $\varepsilon = 0.1$. Find an open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds.

4. Define infinite limits.

P.T.O.

5. State the intermediate value theorem.

6. Show that $\cos\theta$ is continuous at $\theta = 0$.

7. If $y = t(t + 1)(t + 2)$, find $\dfrac{dy}{dt}$ using logarithmic differentiation.

8. Evaluate integral $\displaystyle\int_{\log 2}^{\log 3} e^x dx$.

9. Solve the initial value problem $\dfrac{dy}{dt} = e^t \sin(e^t - 2)$, $y(\log 2) = 0$.

10. Evaluate $\displaystyle\lim_{x\to\infty} \dfrac{\log(x+1)}{\log_2 x}$.

$(5\times1=5)$

Answer **any seven** from the following : **(Weightage 2 each)**

11. Find the derivative of $\sin 2x \cdot \cos 3x$.

12. Evaluate $\displaystyle\int \dfrac{dx}{\sqrt{-x^2 + 4x - 3}}$.

13. Evaluate $\displaystyle\int \dfrac{\operatorname{Sech}\sqrt{t}\,\tanh\sqrt{t}}{\sqrt{t}}\,dt$.

14. Find the nth derivative of $e^{ax}\cos^2 x \sin x$.

15. If $y = \left(\sinh^{-1}x\right)^2$ prove that $\left(1+x^2\right)y_{n+2} + (2n+1)xy_{n+1} + x^2 y_n = 0$.

16. Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.

17. State mean value theorem and verify mean value theorem for the function $f(x) = x^2 + 2x - 1$ on $[0, 1]$.

18. Replace the polar equation $r \sin \theta = 0$ by equivalent Cartesian equations.

19. Find the radius of curvature at (a, 0) on the curve $y^2 = \dfrac{a^2(a-x)}{x}$.

20. Graph the integrand and use area to evaluate the integral $\displaystyle\int_{-2}^{1} |\, x \,|\, dx$.

(7×2=14)

Answer **any three** from the following :

(Weightage 3 each)

21. a) At what point or points in the given interval does the function $f(x) = -3x^2 - 1$ on [0, 1] assume its average value ?

   b) Express the solution of the initial value problem $\dfrac{dy}{dx} = \tan x,\ y(1) = 5$ as an integral.

22. Use Simpson's rule with n = 4 to approximate $\displaystyle\int_{-1}^{1} (x^2 + 1)\, dx$.

23. Prove that :

   i) $\dfrac{\beta(m+1, n)}{\beta(m, n)} = \dfrac{m}{m+n}$

   ii) $\displaystyle\int_{0}^{1} (x \log x)^4\, dx = \dfrac{24}{3125}$.

24. i) Find the areas of the region enclosed by the curves $y = \cos\left(\pi\, ^x\!/_2\right)$ and $y = 1 - x^2$.

   ii) Find the volume of the solid generated by revolving the regions bounded by the curve $y = 4 - x^2$, and the line $y = 2 - x$ about the x-axis.

25. Graph $y = x^{5/3} - 5x^{2/3}$.

(3×3=9)