Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CBCSS – Reg.) Examination, November 2018
(2017 Admn. Only)
CORE COURSE IN MATHEMATICS
3B03MAT – Elements of Mathematics – I

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. State True/False, if C is an infinite set and B is a finite set then C\B a finite set.

2. Define reciprocal equation.

3. State Sturm's theorem.

4. Evaluate the greatest common divisor of –8 and –36.

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Prove that if A is a set with m elements and B is a set with n elements and if $A \cap B = \phi$. Then $A \cup B$ has m+n elements.

6. Let A = {1, 2, 3, 4, 5}, Determine the truth value of the following statements.
   a) $(\exists x \in A)(x + 3 = 10)$　　　　　b) $(\exists x \in A)(x + 3 < 5)$

7. Find a cubic equation with rational coefficients having the roots 2, $1 + \sqrt{2}$.

8. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$ find the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$.

9. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ find the value of $\sum \alpha^2\beta$.

10. Discuss the nature of roots of the equation $x^3 - 8x^2 + 17x - 10 = 0$.

11. Find the sum of the trigonometric series $1 - \dfrac{1}{2}\cos\alpha + \dfrac{1\cdot3}{2\cdot4}\cos 3\alpha + \ldots\ldots$

P.T.O.

12. If a/c & b/c with $\gcd(a, b) = 1$ prove that ab/c.

13. Prove that $\sqrt{2}$ is irrational.

14. Let $n > 1$ and a, b, c positive integers then prove that
   a) $a \equiv a(\bmod\ n)$
   b) $a \equiv b(\bmod\ n),\ b \equiv c(\bmod\ n) \Rightarrow a \equiv c(\bmod\ n)$.

SECTION – C

Answer **any four** questions from among the questions **15** to **20**. These questions carry **4 marks each**.

15. Prove that the set Q of rational numbers is denumerable.

16. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression prove that $2p^3 - 9\,pq + 27r = 0$.

17. Solve the reciprocal equation $60x^4 - 736x^3 + 1433x^2 - 736x + 60 = 0$.

18. Solve the Diophantine equation $172x + 20y = 1000$.

19. Prove that the integer $53^{103} + 103^{53}$ is divided by 39.

20. Using the Sieve of Eratosthenes find all primes not exceeding 50.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6 marks each**.

21. a) State and prove Cantor's theorem.
   b) Show that $p \wedge q$ logically implies $p \leftrightarrow q$.

22. If α, β, γ are the roots of $x^3 - x + 1 = 0$ prove that $\sum \dfrac{1+\alpha}{1-\alpha} = 1$.

23. Solve $x^3 - 2x - 5 = 0$ using carden's method.

24. Using Euclidean algorithm obtain the $\gcd(826, 1890)$ and find integers x and y such that $\gcd(826, 1890) = 826x + 1890y$.