K18U 1899

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2018
(2014–2016 Admissions)
CORE COURSE IN MATHEMATICS
3B03MAT – Elements of Mathematics – I

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Exhibit a bijection between $\mathbb{N}$ and the set of all odd integers greater than 13.

2. Find the equation whose roots are the roots of $x^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0$, with the signs changed.

3. State Sturm's theorem.

4. Give a prime number of the form $n^2 - 4$.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Suppose that S and T are sets such that $T \subseteq S$. If S is a finite set show that T is a finite set.

6. Let $Q(x, y) : x + y = 0$ where $x, y \in \mathbb{R}$. Determine the truth values of the quantifications $\exists y \forall x Q(x, y)$ and $\forall x \exists y Q(x, y)$.

7. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of $\sum \alpha^2\beta^2$.

P.T.O.

8. Solve the equation, $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

9. Find the sum of the reciprocals of the roots of the equation,
   $x^5 + x^2 + 10x + 105 = 0$.

10. Find the number of real roots of the equation, $x^4 - 14x^2 + 16x + 9 = 0$.

11. Find the sum of the trigonometric series, $\frac{\sin\alpha}{1!} + \frac{\sin 2\alpha}{2!} + \frac{\sin 3\alpha}{3!} + \ldots$

12. Prove that the square of any integer is either of the form 3k or 3k + 1.

13. Prove that the product of four consecutive integers is one less than a perfect square.

14. Show that any composite three-digit number must have a prime factor less than or equal to 31.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Show that the set of all rational numbers is denumerable.

16. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, form the equation whose roots are $\beta + \gamma - 2\alpha$, $\gamma + \alpha - 2\beta$, $\alpha + \beta - 2\gamma$.

17. Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

18. Solve : $x^4 - 2x^3 - 12x^2 + 10x + 3 = 0$.

19. Determine all solutions in the positive integers of the diophantine equation, $18x + 5y = 48$.

20. Find the remainder when 2^{50} is divided by 7.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Consider the following propositional functions defined on the domain of all things :

    T(x) : x is a tool, R(x) : x is in the correct place,

    E(x) : x is in excellent condition.

    Write each of the following statements using these propositional functions, quantifiers and logical operations.

    a) Something is not in the correct place.

    b) All tools are in the correct place and are in excellent condition.

    c) Everything is in the correct place and in excellent condition.

    d) Nothing is in the correct place and is in excellent condition.

    e) One of your tools is not in the correct place, but it is in excellent condition.

22. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

23. Solve by Cardan's method : $x^3 - 6x^2 + 3x - 2 = 0$.

24. a) Show that the number of primers is infinite.

    b) If $p \geq 5$ is a prime number, show that $p^2 + 2$ is composite.