K17U 1968

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2017
CORE COURSE IN MATHEMATICS
(2014 Admn. Onwards)
3B03 MAT : Elements of Mathematics – I

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a countable collection of finite sets whose union is not finite.

2. Find the sum of the cubes of the roots of the equation $x^4 + 2x + 3 = 0$.

3. State the fundamental theorem of algebra.

4. Give a prime number of the form $n^3 - 1$. **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. If A_m is a countable set for each $m \in \mathbb{N}$, show that $\bigcup_{m=1}^{\infty} A_m$ is countable.

6. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

7. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, find the value of $\sum \alpha^3\beta$.

8. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression.

P.T.O.

9. Solve the equation $x^3 - 6x^2 + 13x - 10 = 0$, given that the roots are in arithmetic progression.

10. Find the value of k for which $x^3 + 4x^2 + 5x + 2 + k = 0$ has equal roots.

11. Find the condition that all the roots of the equation, $x^3 + px + q = 0$ may be real.

12. Prove that $3a^2 - 1$ is never a perfect square.

13. Prove that the difference of two consecutive cubes is never divisible by 2.

14. Prove or disprove : Every positive integer can be written in the form $p + a^2$, where p is either a prime or 1 and $a \geq 0$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Let C(x) : x has a cat, D(x) : x has a dog and let F(x) : x has a ferret. Express the following statements in terms of C(x), D(x), F(x), quantifiers and logical operators. Let the domain consist of all students in your class.

   a) A student in your class has a cat, a dog and a ferret.

   b) All students in your class have a cat, a dog or a ferret.

   c) Some student in your class has a cat and a ferret, but not a dog.

   d) No student in your class has a cat, a dog and a ferret.

16. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.

17. If α, β, γ be the roots of the equation $x^3 - 6x + 7 = 0$, form an equation whose roots are $\alpha^2 + 2\alpha + 3, \beta^2 + 2\beta + 3, \gamma^2 + 2\gamma + 3$.

18. Find the sum of the trigonometric series, $\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \ldots$

19. Determine all solutions in the integers of the Diophantine equation, $56x + 72y = 40$.

20. Find the remainder when 41^{65} is divided by 7. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Show that the following statements are equivalent :

    a) S is a countable set.

    b) There exists a surjection of $\mathbb{N}$ onto S.

    c) There exists an injection of S into $\mathbb{N}$.

22. Find $\frac{1}{\alpha^5}+\frac{1}{\beta^5}+\frac{1}{\gamma^5}$ where α, β, γ are the roots of the equation $x^3+2x^2-3x-1=0$.

23. Solve : $x^4-3x^2-6x-2=0$.

24. a) Show that the number $\sqrt{2}$ is irrational.

    b) Give an example to show that $a^2 \equiv b^2 \pmod{n}$ need not imply $a \equiv b \pmod{n}$.

(2×6=12)