K16U 2110

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree (CBCSS – Reg./Supple./Imp.) Examination, November 2016 (2014 Admn. Onwards) CORE COURSE IN MATHEMATICS 3B03 MAT : Elements of Mathematics – I

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. These questions carry **1** mark **each**.

1. Determine the number of different surjections from {a, b, c} onto {1, 2}.
2. Form an equation whose roots are the negatives of the roots of the equation, $x^4 - 4x^3 + 6x^2 - x + 2 = 0$.
3. State Sturm's theorem.
4. State the Fundamental Theorem of Arithmetic. **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Give a contrapositive proof for the following theorem.

   If n is an integer and n^2 is even, then n is even.

6. Determine the truth value of the statement, $\forall x\left(x > 0 \rightarrow \exists y\left(\frac{\sqrt{x}}{y} = 3\right)\right)$, if the universe of each variable consists of all integers.

P.T.O.

7. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \alpha^2\beta$.

8. If α, β, γ are the roots of the equation $x^3 - 6x + 7 = 0$, form an equation whose roots are, $\alpha^2 + 2\alpha + 3, \beta^2 + 2\beta + 3, \gamma^2 + 2\gamma + 3$.

9. Solve the equation, $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$, given that one of the root is $1 - \sqrt{5}$.

10. Find the values of 'a' for which the equation, $ax^3 - 9x^2 + 12x - 5 = 0$ has equal roots.

11. Find the sum of the trigonometric series, $\sin x + \sin 2x + \sin 3x + \dots$

12. Show that 41 divides $2^{20} - 1$.

13. If p is a prime and $p|a_1a_2 \dots a_n$, then show that $p|a_k$ for some k, where $1 \le k \le n$.

14. If $ca \equiv cb \pmod{n}$, show that $a \equiv b \pmod{n/d}$, where $d = \gcd(c, n)$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Suppose the variable x represents students and y represents courses. Consider the following propositional functions.

    $C(y)$ = y is a computer science course.

    $F(x)$ = x is a freshman.

    $T(x, y)$ = student x is taking y.

    Write each of the following statements using the above propositional functions and any needed quantifiers and logical operators.

    a) Bob is a freshman.

    b) Every student is taking atleast one course.

    c) Charlie is not taking any courses.

    d) Every freshman is taking a non-computer science course.

16. When f(x) is divided by $x - 1$ and $x + 2$, the remainders are 4 and –2 respectively. Find the remainder when f(x) is divided by $x^2 + x - 2$.

17. Solve the equation, $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that two of its roots are equal in magnitude and opposite in sign.

18. Find the number and position of the real roots and the number of imaginary roots of the equation $x^5 - 5x + 1 = 0$.

19. For $n \geq 1$, show that there are atleast $n + 1$ primes less than 2^{2n}.

20. Find the solutions in positive integers to the Diophantine equation $172x + 20y = 1000$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) Show that a countable union of countable sets is countable.

    b) Prove that the collection $\mathcal{F}(\mathbb{N})$ of all finite subsets of $\mathbb{N}$ is countable.

22. Solve the equation, $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

23. a) Solve the cubic, $x^3 - 18x - 35 = 0$ by Cardon's method.

    b) Show that the equation, $12x^7 - x^4 + 10x^3 - 28 = 0$ has atleast four imaginary roots.

24. Given integers a and b, not both of which are zero, show that there exist integers x and y such that, gcd(a,b) = ax + by. **(2×6=12)**

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