

M 2337

III Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal UI Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2012 COMPLEMENTARY COURSE IN - STATISTICS (For Mathematics/Computer Science) 3C03STA : Standard Distributions

Time: 3 Hours

Max. Weightage: 30

Instructions : Use of calculate and statistical tables permitted.

 $(10 \times 1 = 10)$ 

Answer any 10 questions (weightage 1 each).

1. A random variable X has the following probability mass function.

$$P\left\{x = \frac{(-1)^{k} 2^{k}}{K}\right\} = \frac{1}{2^{k}}; K = 1, 2, 3,$$

Examine the existence of Ex.

2. A continuous random variable X has the density function

$$f(x) = \begin{cases} \frac{2(1-x) & \text{if } 0 < x < 1}{0 & \text{otherwise}} \\ \text{Find E} (3x^2 + 6x) \end{cases}$$

- 3. Define the characteristics function of a random variable. State its important properties.
- 4. State and prove the addition theorem for the expectation of two random variables.
- 5. Two random variables X and Y have the following joint density functions

$$f(x,y) = \begin{cases} \frac{2}{0} & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional mean of x given y = y

6. Define geometric distribution. Derive its mean and variance.

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- 7. Find the moment generating function of exponential distribution.
- 8. Find the quartile deviation of normal distribution.
- 9. A random variable X has the following density function  $f(x) = 60x^2 (1 - x)^3$ ; 0 < x < 1Find the mean and variance of x.
- 10. State the Bernoulli law of large numbers.
- 11. A random variable X has the density functions.

$$f(\mathbf{x}) = \begin{cases} \frac{e^{-\mathbf{x}} ; \mathbf{x} > 0}{0 \text{ otherwise}} \end{cases}$$

Show that the Chehychev's inequality gives.

$$\mathsf{P}\{\mathsf{Ix}\mathsf{-}\mathsf{II}\mathsf{>}2\} < \frac{1}{4}$$

## PART-B

#### (6x2=12)

Answer any 6 questions (Weightage 2 each).

- 12. Show that the pearson's coefficient of correlation between two random variables is not affected by change of origin and scale.
- 13. A random variable X has the following density function.

$$f(x) = \frac{1}{2} e^{-|x|}; -\infty < x < \infty$$

Find its moment generating function.

14. The probability distribution of a random variable X is given as follows :

X: 0 1 2 3

 $p{X = x}: \frac{1}{2} \frac{1}{8} \frac{1}{8} \frac{1}{4}$ 

Find the first three central moments.

- 15. Derive the moment generating function of poisson distribution. Hence find its mean and variance.
- 16. Establish the 'lack of memory' property of geometric distribution.

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- 17. Derive the expression for the even order central moments of normal distribution.
- 18. Define beta distribution of the second kind. Derive its variance.
- 19. State and prove Chehychev's inequality.
- A balanced die is tossed 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

 $(2 \times 4 = 8)$ 

Answer any 2 questions (Weightage 4 each).

- 21. Establish the following relationships
  - i) E [E (X/Y)] = EX
  - ii) E[V(X/Y)] + V[E(X/Y)] = V(X]
  - iii)  $[E \times Y]^2 \le E X^2 E Y^2$
- 22. Derive the recurrence relation for the central moments of binomial distribution. Hence find its first three central moments.
- Define one parameter gamma distribution. Derive its moment generating function and hence establish its additive property.
- 24. State and prove the weak law of large numbers (WLLN) of independent and identically distributed random variables. If  $\{X_n\}$  is a sequence of independent random variables such that.

$$P\left\{X_n = \frac{1}{\sqrt{n}}\right\} = P_n \text{ and } P\left\{X_n = 1 + \frac{1}{\sqrt{n}}\right\} = 1 - P_n$$

Examine whether the weak law of large numbers holds for the sequence {X<sub>n</sub>}.