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M 11245

Reg. No. : ....

Name : ....

III Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Reg./Supple.) Examination; November 2011

COMPLEMENTARY COURSE IN STATISTICS

3C03 STA : Standard Distributions

(Maths/Comp.Sci.)

Time : 3 Hours   Max. Weightage : 30

Instructions : Use of calculator and statistical tables permitted.

PART – A

Answer **any 10** questions (Weightage **1 each**) :   **(10×1=10)**

1. If X is a random variable, show that EX exists, if and only if, $E|X|$ exists.

2. A fair coin is tossed continuously till head appears for the first time. Find the expected number of tosses required.

3. Define the characteristic function of a random variable. Show that it always exists.

4. Establish the additive property of the cumulants of two independents random variables.

5. Show that, with usual notation $E\left[E(X/Y)\right] = EX$.

6. Find the mean and variance of discrete uniform distribution on the first n natural numbers.

7. Find the moment generating function of single parameter gamma distribution.

8. The wages of 1000 workers are normally distributed with mean Rs. 250 and variance Rs. 70. What is the lowest wage of 100 highest paid workers ?

9. Find the mean of the beta distribution of the first kind.

10. State the Bernoulli law of large numbers.

11. If X is the number scored in a throw of a balanced die, show that the Chebychev's inequality gives $P\{|X-3.5|>2.5\}<0.47$.

PART – B

Answer any 6 questions (Weightage 2 each) : **(6×2=12)**

12. Define Pearson's coefficient of correlation r between two random variables. Show that correlation coefficient r always lies between –1 and 1.

13. A random variable X has the following density function :

$$f(x)=\begin{cases} x \text{ if } 0<x<1 \\ 2-x \text{ if } 1<x<2 \\ 0 \quad \text{otherwise} \end{cases}$$

Find the moment generating functions of X.

14. Find the first three central moments of a random variable X with the following density function :

$$f(x)=\begin{cases} 6x(1-x) & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

15. Establish the recurrence relation for the central moments of Poisson distributions.

16. If X and Y are independent Poisson random variables with parameters λ and μ respectively, show that the conditional distribution of X given X + Y is binomial.

17. Find the moment generating function of normal distribution.

18. Define two parameter gamma distribution. Obtain the expression for its mean and variance.

19. State and prove Chebychev's inequality.

20. A fair coin is lossed 400 times. Find an approximate probability that the number of heads lies between 190 and 210.

PART – C

Answer **any two** questions (Weightage **4 each**) : **(2×4=8)**

21. Two random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} 2 - x - y & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

i) The mean and variance of X and Y

ii) The conditional mean of X given Y = y

iii) The correlation coefficient between X and Y.

22. Derive the recurrence relation for the central moments of binomial distribution. Hence find its first three central moments.

23. Define beta distribution of the second kind. Find its mean, variance and harmonic mean.

24. State and prove weak law of large numbers for independent and identically distributed random variables. Examine whether the weak law holds for the sequence $\{X_k\}$ of independent random variables defined as follows :

$$P\{X_k = \pm 2^k\} = \left(1/2\right)^{2k+1},$$

$$P\{X_k = 0\} = 1 - \left(1/2\right)^{2k}$$