K18U 0641

Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree (CBCSS – Regular) Examination, May 2018
(2017 Admission)
CORE COURSE IN MATHEMATICS
2B02MAT : Integral Calculus

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** questions from **1** to **4**. **Each** question carries **1** mark.

1. State the Mean Value Theorem for Definite Integrals.
2. State Fundamental Theorem of Calculus that describes how to evaluate definite integrals.
3. Define Beta Function.
4. Let f be smooth on [a, b]. Define the length of the curve y = f(x) from a to b. **(4×1=4)**

SECTION – B

Answer **any eight** questions among the questions **5** to **14**. **Each** question carries **2** marks.

5. Graph the integrand and use area to evaluate $\int_{-2}^{1} |x|\,dx$.
6. Using the definitions of Hyperbolic sine and Hyperbolic cosine prove that $\int \sinh u\,du = \cosh u + C$.
7. Using reduction formula evaluate $\int \tan^3 x\,dx$.

P.T.O.

8. Evaluate $\int_0^\infty x^3 e^{-x^2}\,dx$.

9. Prove that B(m, n) = B(n, m) where B denotes the Beta Function.

10. The region between the curve $y=\sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

11. Using integration find the length of the curve x = acost, y = asint, $0 \le t \le 2\pi$.

12. Find the area inside one loop of the lemniscate $r^2 = 4\sin 2\theta$.

13. Find the average value f(x, y) = x cos xy over the rectangle R : $0 \le x \le \pi, 0 \le y \le 1$

14. Evaluate $\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)\,dz\,dy\,dx$. **(8×2=16)**

SECTION – C

Answer **any four** questions among the questions **15** to **20**. **Each** question carries **4** marks.

15. Using reduction formula evaluate $\int \sin^2 2\theta \cos^3 2\theta\, d\theta$.

16. Prove that $B(p, q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\,dx$.

17. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid using the Washer method.

18. Find the length of the cardioid $r = 1 - \cos\theta$.

19. Evaluate $\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy$, by reversing the order of integration.

20. Find the volume of the upper region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$. **(4×4=16)**

SECTION – D

Answer **any two** questions among the questions **21** to **24**. **Each** question carries **6** marks.

21. Using the integral for the area as a limit of Riemann sums, find the area of the region between the parabola $y = x^2$ and the x-axis on the interval [0, b].

22. Define Gamma function. State and prove factorial property of the Gamma function. Express $n\Gamma(n)$ in terms of n!.

23. Find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le \frac{1}{2}$ about the x-axis.

24. A thin plate covers the triangular region bounded by the x-axis and the lines $x = 1$ and $y = 2x$ in the first quadrant. The plate's density at the point (x, y) is $\delta(x, y) = 6x + 6y + 6$. Find plate's mass, first moments, center of mass, moments of inertia and radii of gyration about the coordinate axes. **(2×6=12)**
