K17U 1036

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CBCSS – Reg./Supple./Imp.) Examination, May 2017
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. What values of a and b maximize the value of $\int_a^b (x-x^2)\,dx$?

2. Suppose that f is continuous and that

$\int_0^3 f(x)dx = 3$ and $\int_0^4 f(x)dx = 7$. Find $\int_4^3 f(t)dt$.

3. Give an example of improper integral of the third kind.

4. Give the equation of the elliptic paraboloid which is symmetrical with respect to the planes x = 0 and y = 0 and the z-axis and having vertex at the origin. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. They carry **2** marks **each.**

5. Compute the lower sum L and the upper sum u for the function f defined by $f(x) = x^2$ on [0, 1] and $P = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$.

P.T.O.

6. Show that the value of $\int_0^1 \sqrt{1+\cos x}\, dx$ cannot possibly be 2.

7. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x; -1 \le x \le 2$.

8. Test for convergence : $\int_{-\infty}^{\infty} \frac{x^3 + x^2}{x^6 + 1}\, dx$.

9. Given $\int_0^{\infty} \frac{x^{p-1}}{1+x}\, dx = \frac{\pi}{\sin p\pi}$, show that $\Gamma(p)\,\Gamma(1-p) = \frac{\pi}{\sin p\pi}$ where $0 < p < 1$.

10. Find the area of the region enclosed between $y = x^2$ and $y = -x^2 + 4x$.

11. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

12. Find the length of the curve $y = \int_0^x \sqrt{\cos 2t}\, dt$ from $x = 0$ to $x = \pi/4$.

13. Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dx\, dy$.

14. Find the area of the region cut from the first quadrant by the curve $r = 2(2 - \sin 2\theta)^{1/2}$.

(2×8=16)

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. They carry **4** marks **each**.

15. Show that for any positive integer n, $\int (\ln x)^n\, dx = x(\ln x)^n - n \int (\ln x)^{n-1}\, dx$.

16. Prove that $\int_0^1 x^m (\ln x)^n\, dx = \frac{(-1)^n\, n!}{(m+1)^{n+1}}$ where n is a positive integer and $m > -1$.

17. The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = \frac{3}{2}$ to generate a solid. Find the volume of the solid.

18. Find the area of the surface generated by revolving the curve $y = x^3$; $0 \le x \le \frac{1}{2}$ about the x-axis.

19. Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top his in the plane $z = f(x, y) = 3 - x - y$.

20. Evaluate $\iint\limits_R e^{x^2+y^2}\, dy\, dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. They carry **6** marks **each**.

21. Show that $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a}$.

22. Prove that $2^{2p-1}\,\Gamma(p)\,\Gamma(p+\frac{1}{2}) = \sqrt{\pi}\,\Gamma(2p)$.

23. Find the area of the surface swept out by revolving the circle of radius 1 centred at the point (0, 1) about the x-axis.

24. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\, dy\, dx$. **(6×2=12)**

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