K16U 1221

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2016
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. The set P = {0, .2, 6, 1, 1.5, 2} is a partition of [0, 2]. Find || P ||.

2. If f(x) has the constant value c on [a, b], find $\int_a^b f(x)\, dx$.

3. Give an example of improper integral of the second kind.

4. Give the equation of the quadric surface known as hyperboloid of one sheet.

(4×1=4)

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. They carry **2** marks **each**.

5. Express the limit of Riemann sums $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(C_k^2 - 3C_k\right)\Delta x_k$, where P is a partition of [–7, 5].

6. Obtain the upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2}\, dx$ given by the max-min inequality.

P.T.O.

7. Express the solution of the following initial value problem interms of integrals,

$\frac{dy}{dx} = \sec x,\ y(2) = 3.$

8. Examine for convergence : $\int_1^{\infty} \frac{\ln x}{x + a}\, dx$, where a is a positive constant.

9. Evaluate the integral, $\int_0^{\infty} x^6 e^{-2x}\, dx$.

10. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

11. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$; $1 \leq y \leq 4$ about the y-axis.

12. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$; $1 \leq x \leq 2$ about the x-axis.

13. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy\, dx$ and write an equivalent integral with the order of integration reversed.

14. Find the average value of $f(x, y) = x \cos xy$ over the rectangle $R : 0 \leq x \leq \pi,\ 0 \leq y \leq 1$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. They carry **4** marks **each.**

15. Show that the function f defined by $f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$ has no Riemann integral over [0, 1].

16. Show that $\int_0^2 x \sqrt[3]{8 - x^3}\, dx = \frac{16\pi}{9\sqrt{3}}$.

17. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

18. Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

19. Find the average value of $F(x, y, z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$ in the first octant.

20. Evaluate $\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x-y}{2}\, dx\, dy$

by applying the transformation $u = \frac{2x-y}{2}$, $v = y/2$ and integrating over an appropriate region in the uv-plane. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. They carry **6** marks **each**.

21. Find $\int \sin^2 x \cos^3 x\, dx$.

22. Prove that $\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$.

23. Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$; $0 \leq t \leq 2\pi$.

24. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$. **(2×6=12)**
