



K20U 0468

Reg. No. : .....

Name : .....



II Semester B.Sc. Degree (CBCSS (OBE) Regular) Examination, April 2020  
(2019 Admission)

Core Course in Mathematics  
2B02 MAT : INTEGRAL CALCULUS AND LOGIC

Time : 3 Hours

Max. Marks : 48

PART – A

Short Answer Questions – Answer any 4.

1. Evaluate  $\int_0^{\frac{\pi}{2}} \sin^5 x \, dx$ .

2. Express the equation  $x + y = 0$  in polar form.

3. Express the cartesian coordinates  $(x, y, z)$  in terms of the spherical coordinates  $(\rho, \theta, \phi)$ .

4. Give an example for a sentence which is not a statement.

5. Check the truthness/falsity of the statement  $(\forall n \in \mathbb{P}) (n + 2 > 3)$ , where  $\mathbb{P}$  is the set of all natural numbers. Justify your answer. (4×1=4)

PART – B

Short Essay Questions – Answer any 8.

6. Find  $\frac{d}{dx}(\operatorname{sech}^2(x^3))$ .

7. Evaluate  $\int e^{-x} \sinh 2x \, dx$ .

8. Evaluate  $\int_0^3 \int_0^4 \frac{e^{xy}}{y} \, dy \, dx$ .

9. Evaluate  $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r \, dr \, d\theta$ .

10. Find the area of a disc of radius  $a$  using double integral.

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11. Find  $\int_0^4 x^2 dx$  using trapezoidal rule, taking two subintervals.

12. Find  $\int_0^4 f(x) dx$  using Simpson's 1/3 rule, where the function  $f(x)$  is given by

x	0	1	2	3	4
f(x)	1	2	4	7	9

13. Explain the terms :

- i) Conjunction of two statements
- ii) Contrapositive of an implication.

14. If  $n$  is an odd integer, prove that  $n^2 + 1$  is an even integer.

15. Write the converse and contrapositive of the statement : if  $x \geq y$ , then  $x - y < 0$ .

16. Show that if  $m$  and  $n$  are natural numbers and  $m + n = 10$ , then either  $m \leq 5$  or  $n \leq 5$ .

(8×2=16)

### PART – C

Essay Questions – Answer any 4.

17. Evaluate  $\int_0^{\frac{\pi}{4}} \sin^3 4\theta \cos^2 2\theta d\theta$ .

18. Derive the reduction formula for  $\int \tan^n x dx$ ,  $n > 1$ .

19. Convert into polar form and evaluate  $\int_0^3 \int_x^3 \frac{y^2}{x^2 + y^2} dy dx$ .

20. Evaluate  $\int_0^1 \int_0^{2x} \int_{x-z}^{x+z} (x^2 + 2z) dV$ .

21. Explain the Simpson's 1/3 rule of integration.

22. Prove the following statement using the method of contradiction : "If  $n$  is an integer and  $n^2$  is odd, then  $n$  must be odd". Is the converse true ? Justify.

23. Write the negation of the following statements using quantifiers.

- i)  $(\forall x \in \mathbb{R}) (2x < 7)$
- ii)  $(\exists x \in \mathbb{R}) (2x > 7)$ .

(4×4=16)



PART – D

Long Essay Questions – Answer **any 2**.

24. Derive reduction formula for  $\int \sin^m x \cos^n x \, dx$ , where  $m, n > 1$  and use it to

evaluate  $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x \, dx$ .

25. Use triple integration in spherical coordinates to find the volume of the solid  $V$  bounded above by the sphere  $x^2 + y^2 + z^2 = 4$  and bounded below by the cone  $z = \sqrt{x^2 + y^2}$ .

26. Evaluate  $\int_0^6 \frac{1}{2x+1} \, dx$  using Simpson's 1/3 rule, taking six subintervals.

27. a) Prove that if  $x$  is a real number greater than 1, then  $x^2 > x$ .

b) Test the truthness of the following statements (with justifications) :

i)  $(\forall x \in \mathbb{R}) (x^2 \geq x)$ , where  $\mathbb{R}$  is the set of all real numbers.

ii)  $(\forall x \in \mathbb{Z}) (x^2 \geq x)$ , where  $\mathbb{Z}$  is the set of all integers.

(2×6=12)

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