K20U 0310

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CBCSS – Supplementary/Improvement) Examination, April 2020
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus
(2017-2018 Admissions)

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** questions from **1** to **4**. **Each** question carries **1** mark.

1. State Fundamental Theorem of Calculus (Part 2).
2. State the Mean Value Theorem for Definite Integrals.
3. Define Gamma function.
4. Let f be smooth on [a, b]. Define the length of the curve y = f(x) from a to b.

(4×1=4)

SECTION – B

Answer **any eight** questions among the questions **5** to **14**. **Each** question carries **2** marks.

5. Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$ and the x-axis.
6. Using the definitions of Hyperbolic sine and Hyperbolic cosine prove that $\int \sinh u \, du = \cosh u + C$.
7. Derive the reduction formula for $\int (\ln x)^n \, dx$.
8. Evaluate $\int_0^1 \left(\ln \frac{1}{y}\right)^{n-1} dy$ using the definition of Gamma Function.
9. Prove that B(m, n) = B(n, m) where B denotes the Beta Function.
10. Using integration find the length of the curve $x = a \cos t$, $y = a \sin t$, $0 \leq t \leq 2\pi$.
11. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}, 1 \leq y \leq 4$ about y-axis.

P.T.O.

12. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the xy-plane is $x = \cos t$, $y = 1 + \sin t$, $0 \le t \le 2\pi$. Using this parametrization find the area of the surface swept out by revolving the circle about the x-axis.
13. Evaluate $\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)\,dz\,dy\,dx$.
14. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$. **(8×2=16)**

SECTION – C

Answer **any four** questions among the questions **15** to **20**. **Each** question carries **4** marks.

15. Prove that $\sinh^{-1} x = \ln\left(x+\sqrt{x^2+1}\right), -\infty < x < \infty$.
16. Prove that $B(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\,dx$.
17. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid using the washer method.
18. Find the length of the cardioid $r = a(1 - \cos\theta)$.
19. Evaluate $\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy$, by reversing the order of integration.
20. A solid of constant density $\delta = 1$ occupies the region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$. Find the solid's moment of inertia about the z-axis, using triple integrals in spherical coordinates. **(4×4=16)**

SECTION – D

Answer **any two** questions among the questions **21** to **24**. **Each** question carries **6** marks.

21. Using the integral for the area as a limit of Riemann sums, find the area of the region between the parabola $y = x^2$ and the x-axis on the interval [0, b].
22. Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
23. Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.
24. A thin plate covers the triangular region bounded by the x-axis and the lines $x = 1$ and $y = 2x$ in the first quadrant. The plates density at the point (x, y) is $\delta(x, y) = 6x + 6y + 6$. Find plate's mass, first moments, center of mass, moments of inertia and radii of gyration about the coordinate axes. **(2×6=12)**

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