K20U 0309

Reg. No. : .....

Name : .....

II Semester B.Sc. Degree (CBCSS – Supplementary) Examination, April 2020

CORE COURSE IN MATHEMATICS

2B02 MAT : Integral Calculus

(2014-2016 Admissions)

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Find the derivative of $y = \tan h \sqrt{1+t^2}$.
2. Find the value of $\Gamma(1)$.
3. Write the formula for the length of a smooth curve $x = g(y)$, $c \le y \le d$.
4. Evaluate $\int_0^1 \int_0^1 \int_0^2 dx\, dy\, dz$.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Express $\lim_{\|P\| \to 0} \sum_{k=1}^{n} (C_k^2 - 3C_k)\, \Delta x_k$, where P is a partition of [–7, 5], as a definite integral.
6. If f is continuous on [a, b] $a \neq b$ and if $\int_a^b f(x)\,dx = 0$ then show that $f(x) = 0$ at least once in [a b].
7. Evaluate $\int \frac{2\,dx}{\sqrt{3+4x^2}}$.
8. Show that $\int_1^\infty \frac{dx}{x^2}$ converges to 1.
9. Express the integral $\int_0^1 x^4(1-x)^3 dx$ as a β-function and hence find its value.

P.T.O.

10. Write the equation of the hyperboloid of one sheet and its section by $z = 0$, $x = 0$, $y = 0$.

11. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \le y \le 4$ about y-axis.

12. Using the parametrization $x = \cos t$, $y = 1 + \sin t$ $0 \le t \le 2\pi$, of the circle of radius 1 centered at the point (0, 1), find the area of the surface swept out by revolving the circle about x-axis.

13. Evaluate $\int_{1}^{2}\int_{y}^{y^2} dx\, dy$.

14. Evaluate $\int_{0}^{1}\int_{0}^{3-3x}\int_{0}^{3-3x-y} dz\, dy\, dx$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find $\int x \sin^{-1} x\, dx$.

16. Evaluate $\int_{0}^{\infty} x^5 \cdot e^{-2x}\, dx$.

17. Find the surface area of the solid generated by revolution of the curve $y = 2\sqrt{x}$, $1 \le x \le 2$ about the x-axis.

18. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

19. Evaluate $\iint_{R} e^{x^2+y^2}\, dy\, dx$ where R is the semicircular region bounded by the x- axis and the curve $y = \sqrt{1-x^2}$.

20. Evaluate $\int_{0}^{1}\int_{0}^{1-x} \sqrt{x+y}\,(y-2x)^2\, dy\, dx$.

SECTION – D

Answer **any two** questions from **21** to **24**. These questions carry **6** marks **each**.

21. Find
    i) $\int \tan^5 x \, dx$
    ii) $\int (\ln x)^n \, dx$.

22. Prove that $B(u, v) = \frac{\Gamma(u) \cdot \Gamma(v)}{\Gamma(u+v)}$, $u, v > 0$.

23. Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \frac{1}{2}$ about x-axis.

24. Find the volume of the region D enclosed by the surface $Z = x^2 + 3y^2$ and $Z = 8 - x^2 - y^2$.