K19U 0265

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2019
(2017 Admission Onwards)
Core Course in Mathematics
2B02MAT : INTEGRAL CALCULUS

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** questions from **1** to **4**. **Each** question carries **1** mark.

1. Find $\Gamma(1)$, where Γ is the Gamma function.
2. State the formula for the length of a smooth curve $x = g(y)$, $c \leq y \leq d$.
3. State Fubini's Theorem (First form).
4. Define the average value of a function F over a region D in space. **(4×1=4)**

SECTION – B

Answer **any eight** questions among the questions **5** to **14**. **Each** question carries **2** marks.

5. Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$ and the x-axis.
6. Derive the reduction formula for $\int (\ln x)^n dx$.
7. Evaluate $\int_0^1 \left(\ln\frac{1}{y}\right)^{n-1} dy$ using the definition of Gamma function.

P.T.O.

8. Using the definition of Beta function, evaluate $\int_0^1 x^4(1-x)^3 dx$.

9. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \le y \le 4$, about y-axis.

10. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the xy-plane is $x = \cos t$, $y = 1 + \sin t$, $0 \le t \le 2\pi$. Using this parametrization find the area of the surface swept out by revolving the circle about the x-axis.

11. Using integration find the area enclosed by the polar curve $r = a$, $0 \le \theta \le 2\pi$, a is a constant.

12. Evaluate $\int_0^3 \int_0^2 (4-y^2)dydx$.

13. Using triple integral find the volume of the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$, $z = 2$ in the first octant.

14. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$. **(8×2=16)**

SECTION – C

Answer **any four** questions among the questions **15** to **20**. **Each** question carries **4** marks.

15. State and prove the Mean value theorem for definite integrals.

16. Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, $-\infty < x < \infty$.

17. Prove that $\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$ where Γ is the Gamma function.

18. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$ about x-axis.

19. Find the perimeter of the cardioid $r = a(1 - \cos\theta)$.

20. A solid of constant density $\delta = 1$ occupies the region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$. Find the solid's moment of inertia about the z-axis, using triple integrals is spherical coordinates. **(4×4=16)**

SECTION – D

Answer **any two** questions among the questions **21** to **24**. **Each** question carries **6** marks.

21. State and prove the Fundamental Theorem of Calculus (Part 2).

22. Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

23. Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.

24. Find the centroid of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below by the xy-plane (given $\delta = 1$). **(2×6=12)**
