K19U 0264

Reg. No. : .....

Name : .....

II Semester B.Sc. Degree (CBCSS – Supple.) Examination, April 2019
(2014-'16 Admissions)
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. State the fundamental theorem of calculus (Part II).
2. Classify the type of the improper integral $\int_0^{\infty} \frac{dx}{1+\tan x}$.
3. Write down the formula for finding the length of a smooth curve $x = g(y)$, $c \leq y \leq d$.
4. Evaluate $\int_0^1 \int_0^1 \int_0^2 dx\,dy\,dz$.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Express the limit of Reimann sum $\lim_{\|P\|\to 0} \sum_{k=1}^{n} (3C_k^2 - 2C_k + 5)\Delta x_k$ as an integral if P denotes a partition of the interval [– 1, 3].
6. If 'f' is continuous on [a, b], $a \neq b$ and if $\int_a^b f(x)\,dx = 0$, then show that $f(x) = 0$ atleast once in [a, b].

P.T.O.

7. Evaluate $\int_0^{2\sqrt{3}} \frac{dx}{\sqrt{4+x^2}}$.

8. Show that $\int_0^{\infty} \frac{\cos x}{x^2+1}$ is absolutely convergent.

9. Show that B(u, v) = B(v, u), where B(u, v) is the Beta function.

10. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}} - 1$ where $0 \leq x \leq 1$.

11. Using the parametrization x = cost, y = 1 + sin t, $0 \leq t \leq 2\pi$, of the circle of radius 1 centered at the point (0, 1), find the area of the surface swept out by revolving the circle about the x-axis.

12. Find the area of the region in the plane enclosed by the cardioid $r = 2(1+\cos\theta)$.

13. Find the average value of f(x, y) = x cos xy over the rectangle R, $0 \leq x \leq \pi$, $0 \leq y \leq 1$.

14. Evaluate $\int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{2-r^2}} 3\, dz\, dr\, d\theta$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$ and the x-axis.

16. Test the convergence of $\int_1^{\infty} \frac{x}{3x^4+5x^2+1} dx$.

17. Find the area of the surface generated by revolutions of the curve $y = \frac{x^3}{9}$, $0 \leq x \leq 2$ about x-axis.

18. Find the length of one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

19. Calculate $\iint_R \frac{\sin x}{x} dA$, where R is the triangle in the xy plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.

20. A solid of constant density $\delta = 1$ occupies the upper region D cut from the solid $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$. Find the solid moment of inertia about z-axis.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \leq x \leq 2$.

22. Show that $\left|\left(\frac{1}{2}\right) = \sqrt{\pi}\right.$.

23. Find the perimeter of the cardioid $r = a(1 - \cos\theta)$.

24. Find the centroid of the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$.
