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K19U 3184

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS- Supplementary/Improvement) Examination, November-2019

(2017 -2018 Admissions)

CORE COURSE IN MATHEMATICS

1B01 MAT: DIFFERENTIAL CALCULUS

Time : 3 Hours

Max. Marks :48

SECTION-A

I. All the first **Four** questions are compulsory. They carry **1** mark each. **(4×1=4)**

1. Find $\lim_{y \to -2} \frac{y+2}{y^2+5y+6}$.

2. Find the value of $\cosh x$ if $\sinh x = \frac{4}{3}$

3. Find the Cartesian coordinate corresponding to $(-3, \pi)$.

4. Find an equation for the circular cylinder $4x^2+4y^2=9$ in cylindrical coordinates.

SECTION - B

II. Answer any **Eight** questions from among the questions **5** to **14.** These questions carry 2 marks each. **(8×2=16)**

5. Evaluate $\lim_{u \to 1} \frac{u^4-1}{u^3-1}$.

6. Show that $f(x)=\frac{x^2+x-6}{x^2-4}$ has a continuous extension to $x=2$, and

P.T.O.

find that extension.

7. Find the derivative of $y = \sinh^{-1}(\tan x)$ with respect to x.

8. Find the spherical coordinate equation for $x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4}$.

9. Calculate $\frac{dS}{d\theta}$ for $r = a(1 - \cos\theta)$.

10. Find the radius of curvature of the parabola $y^2 = 4ax$ at $(at^2, 2at)$

11. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{1 + \tan x}$

12. Find the maximum and minimum values of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $(0,2)$.

13. Find an equation for the level surface of the function $f(x,y,z) = \ln(x^2 + y + z^2)$ through (-1,2,1).

14. Show that the function $f(x,y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x,y) approaches (0,0).

SECTION - C

III. Answer any **Four** questions from among the questions **15 to 20.** These questions carry **4** marks each. **(4×4=16)**

15. If $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

16. For the cardiod $r = a(1 + \cos\theta)$ show that $\frac{\rho^2}{r}$ is constant.

17. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct

to 4 decimal places.

18. Verify Langrange's mean value theorem for $f(x)=(x-1)(x-2)(x-3)$ in $(0,4)$ and find appropriate value for c.

19. Express $\frac{\partial \omega}{\partial r}$ and $\frac{\partial \omega}{\partial s}$ in terms of r and s,

    if $\omega = x + 2y + z^2, x = \frac{r}{s}, y = r^2 + \ln s, z = 2r$.

20. Verify Euler's theorem for $z=(x^2+xy+y^2)^{-1}$.

SECTION - D

IV. Answer any **Two** questions from among the questions **21** to **24**. These questions carry **6** marks each. **(2×6=12)**

21. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ .Hence find the value of y_n when $x=0$.

22. Find the coordinates of the center of curvature at the point x=at^2, $y=2at$. on the parabola $y^2=4ax$ and hence find its evolute.

23. Find the volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius a.

24. If $u=\tan^{-1}x\left(\frac{x^3+y^3}{x-y}\right), x \neq y$ show that

    (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

    (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u$.
