K18U 2184

Reg. No. : .....

Name : .....

I Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, November 2018
CORE COURSE IN MATHEMATICS
1B01 MAT : Differential Calculus
(2017 Admn. Onwards)

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Find $\lim_{x \to -1} \frac{x^3 + 4x^2 - 3}{x^2 + 5}$.
2. Find the value of cosh x if $\sinh x = \frac{-3}{4}$.
3. Find the Cartesian equation for $r = \frac{4}{2\cos\theta - \sin\theta}$.
4. Find an equation for the cylinder $x^2 + (y-3)^2 = 9$ in cylindrical co-ordinates. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Find $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3} - 2}$.
6. If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \to 4} f(x)$.
7. Find the derivative of $y = \sinh^{-1} \sqrt{x}$ with respect to x.
8. Translate the equation $x^2 + y^2 + (z-1)^2 = 1$ into a spherical co-ordinate equation.

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9. Calculate $\frac{dS}{dx}$ for $ay^2 = x^3$.

10. Find the co-ordinate of the center of curvature at any point of the parabola $y^2 = 4ax$.

11. Verify Rolle's theorem for $\frac{\sin x}{e^x}$ in $(0, \pi)$.

12. Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

13. Find the domain and range of the function $f(x, y) = \sin^{-1}(y - x)$.

14. Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is continuous at every point except at the origin. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. If $y = e^{ax} \sin bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2) y = 0$.

16. Prove that radius of curvature at any point of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $3a \sin t \cos t$.

17. Using Maclaurin's series, expand $\tan^{-1}x$.

18. Verify Langrange's mean value theorem for $f(x) = (x - 1)(x - 2)(x - 3)$ in $(0, 4)$ and find appropriate value for c.

19. Using chain rule find $\frac{dw}{dt}$, where $w = xy$, $x = \cos t$, $y = \sin t$ at $t = \frac{\pi}{2}$.

20. Verify Euler's theorem for $z = ax^2 + 2hxy + by^2$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0$. Hence find $(y_n)_0$.

22. Find the evolute of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$.

23. A rectangular sheet of metal of length 6 meters and width 2 meters is given. Four equal squares are removed from the corners. The sides of this sheet are now turned up to form an open rectangular box. Find approximately, the height of the box, such that the volume of the box is maximum.

24. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $x \neq y$ show that

    i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

    ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \left(1 - 4\sin^2 u\right)\sin 2u$. **(6×2=12)**
