K17U 2587

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – Reg.) Examination, November 2017
CORE COURSE IN MATHEMATICS
1B01 MAT : Differential Calculus
(2017 Admn.)

Time : 3 Hours | Max. Marks : 48

SECTION – A

All the **4** questions are **compulsory**. **Each** question carries **1** mark.

1. State Leibnitz's theorem.
2. Find the derivative of sinhx.
3. Verify Rolle's theorem for the function $16x - x^2$.
4. Define interior point in a region. **(1×4=4)**

SECTION – B

Answer **any 8** questions. **Each** question carries **2** marks.

5. Find $\lim_{x \to 2^+} \frac{x^2 - 1}{2x + 4}$.
6. Prove that $\cosh^2 x - \sinh^2 x = 1$.
7. Find the 3^{rd} derivative of $3\sin x + 5\cos x$.
8. Find the radius of curvature at (x, y) for the curve $a^2 y = x^3 - a^3$.
9. Find the polar equation of the circle $x^2 + (y - 3)^2 = 9$.
10. Find the spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$.
11. Expand sec x using Maclaurin's theorem.
12. Find the asymptotes of $x^3 + y^3 = 3axy$.
13. Find an equation for the level surface of the function $f(x, y, z) = \sqrt{x - y} - \ln z$.
14. Verify Euler's theorem for the function $u = x^2 + y^2$. **(2×8=16)**

P.T.O.

SECTION – C

Answer **any 4** questions. **Each** question carries **4** marks.

15. For what values of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$ is continuous at every x.

16. Evaluate $\int \frac{\operatorname{sech}\sqrt{t}\tanh\sqrt{t}\,dt}{\sqrt{t}}$.

17. Show that the evolute of the parabola $x^2 = 4ay$ is $4(y - 2a)^3 = 27ay^2$.

18. Using L'Hospital's Rule, find $\lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$.

19. Find the points of inflexion on the curve $y = 3x^4 - 4x^3 + 1$.

20. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$. **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **6** marks.

21. If $y = e^{m\cos^{-1}x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$.

22. Find the radius of curvature at the point (1, 1) on the curve $x^3 + y^3 = 2xy$.

23. State Taylor's theorem. Prove that $ln \cosh x = \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} - \ldots$

24. If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, then show that z is a homogeneous function.

    Also verify the Euler's theorem. **(6×2=12)**
