K17U 2542

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (C.B.C.S.S. – Reg./Supple./Improv.) Examination, November 2017
CORE COURSE IN MATHEMATICS
1B01 MAT : Differential Calculus (2014 – 16 Admns.)

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Evaluate $\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$.

2. For what value of a is $f(x)=\begin{cases} x^2-1 & x<3 \\ 2ax & x\geq 3\end{cases}$ is continuous at every x ?

3. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x+x^2}$.

4. State Rolle's theorem. **(1×4=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Given f(x) = 5 – 4x, find $f^{-1}(x)$ and evaluate df^{-1}/dx at x = f(1/2).

6. Find the equation of the sphere which has its centre at the origin and which passes through the point (2, 3, 6).

7. Find $\lim_{x\to\infty} x^{1/x}$.

P.T.O.

8. Obtain the Maclaurin series for $e^{\sin x}$ upto the term containing x^2.

9. Find the radius of curvature at any point of the curve : $x = 2t$, $y = t^2 - 1$.

10. Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.

11. Show that $f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & \text{if}(x, y) \neq (0, 0) \\ 0, & \text{if}(x, y) = (0, 0) \end{cases}$ is discontinuous at the origin.

12. Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.

13. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point $(3, 2)$.

14. If $u = ze^{ax+by}$, where z is a homogeneous function in x and y of degree n, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (ax + by + n)u$. **(2×8=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15\. a) Find the derivative of $y = \cos^{-1} x - x \operatorname{sech}^{-1} x$ with respect to x.

b) Find $\lim_{x \to 0} x^2 e^{\frac{1}{\sin x}}$.

16. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$.

17. In spherical polar coordinates, a certain surface is described by the equation $\rho = 2\cos\phi$. Find its equation in Cartesian coordinates and cylindrical coordinates.

18. Find the inflection point of the curve $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3}$. Identify the intervals on which it is concave up and concave down.

19. For $g(x) = \frac{x-4}{x-3}$, decide if we can use Lagrange's mean value theorem on [4, 6]. If so, find c. If not, explain why.

20. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. Hence find the value of y_n when $x = 0$.

22. Find the equation of the right circular cone whose vertex is at the origin and which passes through the straight lines

$$\frac{x}{3} = \frac{y}{6} = \frac{z}{-2};\quad \frac{x}{2} = \frac{y}{2} = \frac{z}{-1};\quad \frac{x}{-1} = \frac{y}{2} = \frac{z}{2}.$$

Find the axis and the semivertical angle of the cone.

23. Show that the right circular cylinder of given surface (including the ends) and maximum volume is such that its height is equal to the diameter of the base.

24. If $V = \log_e \sin\left[\frac{\pi(2x^2 + y^2 + xz)^{1/2}}{2(x^2 + xy + 2yz + z^2)^{1/3}}\right]$, find the value of $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}$ when $x = 0, y = 1, z = 2$. **(6×2=12)**
