



K16U 2499

Reg. No. :

Name :

I Semester B.Sc. Degree (C.C.S.S. – Reg./Supple./Improv.)

Examination, November 2016

CORE COURSE IN MATHEMATICS

1B01 MAT : Differential Calculus

(2014 Admn. Onwards)

Time : 3 Hours

Total Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry 1 mark each.

1. State Sandwich Theorem.
2. The derivative of $\sin^{-1}(x^2)$ is _____
3. True or false : If $f'(a) = 0$ and $f''(a)$ is negative then $f(x)$ is maximum at $x = a$.

4. Find $\lim_{(x,y) \rightarrow (3,4)} \sqrt{x^2 + y^2 - 1}$. (4×1=4)

SECTION – B

Answer **any 8** questions from among the questions 5 to 14. They carry 2 marks each.

5. Find $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$.

6. Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x .

7. If $y = e^{ax} \sin bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

8. Replace the polar equation $r^2 = 4r \cos \theta$ by equivalent Cartesian equation and identify the graph.

P.T.O.



9. Graph the set of points whose polar coordinates satisfy the inequalities

$$0 \leq r \leq 1 \text{ and } \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}.$$

10. In the Mean Value Theorem $f(b) - f(a) = (b - a) f'(c)$. Determine 'c' lying between

$$a \text{ and } b \text{ if } f(x) = x(x - 1)(x - 2), a = 0 \text{ and } b = \frac{1}{2}.$$

11. Find $\frac{ds}{dx}$ if $y = c \cosh(x/c)$.

12. Find $\lim_{x \rightarrow \infty} \frac{\ln x}{2\sqrt{x}}$.

13. Find the domain and range of the function $w = \frac{1}{xy}$.

14. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

(8×2=16)

SECTION - C

Answer **any 4** questions from **15 to 20**. They carry **4** marks **each**.

15. If $ax^2 + 2hxy + by^2 = 1$, find $\frac{d^2y}{dx^2}$.

16. Show that $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ has a continuous extension to $x = 2$ and find that extension.

17. Find 'ρ' at the origin for the curve $y^4 + x^3 + a(x^2 + y^2) - a^2y = 0$.

18. Expand $e^{\sin x}$ by Maclaurin's series.

19. If $u = \frac{x^2y^2}{x^2 + y^2}$, show that $\frac{x\partial^2u}{\partial x\partial y} + \frac{y\partial^2u}{\partial y^2} = \frac{\partial u}{\partial y}$.

20. Find a linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2). (4×4=16)



SECTION - D

Answer **any 2** questions from **21 to 24**. They carry **6** marks **each**.

21. If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + a^2) y_n = 0$.
Hence find the value of y_n when $x = 0$.

22. a) Find an equation for the hyperbola with excentricity $3/2$ and directrix $x = 2$.

b) Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

c) Find the Cartesian coordinate of the point $(-3, 2\pi)$.

23. Show that the evolute of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is another equal cycloid.

24. If $\sin V = \frac{(x + 2y + 3z)}{\sqrt{x^2 + y^2 + z^2}}$, show that $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} + z \frac{\partial V}{\partial z} + 3 \tan V = 0$.

(2×6=12)