K24U 2750

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular /Supplementary/ Improvement) Examination, November 2024
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **any four** questions from this Part. **Each** question carries **1** mark **each.** **(4×1=4)**

1. Does the set $S = \{1, 4, 9, 16, ...\}$ is denumerable ? Justify your answer.
2. Find the cubic equation whose roots are $1, -1, 2$.
3. If α, β are the roots of the equation $x^2 - 5x + 17 = 0$. Write down the value of $\alpha^2 + \beta^2$.
4. Show that 2 is a double root of the equation $x^3 - 4x^2 + 4x = 0$.
5. Find $\arg(Z)$ if $Z = 1 + i$.

SECTION – B

Answer **any eight** questions from the following. **Each** question carries **2** marks. (8×2 =16)

6. Show that the set of all negative integers is countable.
7. If $1/\alpha, \beta, \gamma$ are the roots of the equation $x^3 + ax^2 + bx + c = 0$. Find the equation whose roots are $1/\alpha, 1/\beta, 1/\gamma$.
8. Solve $2x^3 + x^2 - 7x - 6 = 0$, given that difference between two of the roots is 3.

P.T.O.

9. Solve $4x^3 - 24x^2 + 23x + 18 = 0$ given that the roots are in arithmetical progression.

10. q, r, s are positive. Show that the equation $x^4 + qx^2 + rx - s = 0$ has one positive, one negative and two imaginary roots.

11. State The Descarte's rule of signs.

12. State the general form of De Movier's Theorem.

13. Solve the equation $x^3 = 1$.

14. Using De moviers theorem find $(1+i)^4$.

15. Does the equation $2x^2 - 5x + 2 = 0$ is a reciprocal equation ? Justify your answer.

16. Given that ω is a cube root of unity, Show that $\omega^2 + \omega + 1 = 0$

SECTION – C

Answer **any four** questions. **Each** question carries 4 marks **each**. **(4×4=16)**

17. Show that the set of real numbers R is uncountable.

18. Show that every equation of n^{th} degree has exactly n roots.

19. Prove the following : If $\alpha_1, \alpha_2, ..., \alpha_n$ are the roots of the equation $x^n + p_1x^{n-1} + p_2x^{n-2} + ... + p_n = 0$ then sums of the products of $\alpha_1, \alpha_2, ..., \alpha_n$ taken one, two, ..., n at a time, are respectively equal to $-p_1, p_2, ..., (-1)^n p_n$.

20. α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Hence write down the value of $\Sigma(1 + \alpha)/(1 - \alpha)$.

21. Solve $4x^4 - 4x^3 - 13x^2 + 9x + 9 = 0$, given that sum of two roots is zero.

22. If all the roots of $ax^3 + 3bx^2 + 3cx + d = 0$ are real, show that the equation can be reduced to $t^3 - t + \mu = 0$, where $27\mu^2 < 4$, by a substitution of the form $x = p + qt$, where p and q are real.

23. Find the value of $\sqrt{-8-6i}$.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks **each**. **(2×6 =12)**

24. a) Given that A and B are countable sets. Show that $A \cup B$ is countable.

 b) State Cantor's Theorem.

25. Find the rational roots of the equation $6x^4 - 25x^3 + 26x^2 + 4x - 8 = 0$.

26. Transform the equations

 a) $x^3 - 6x^2 + 4x - 7 = 0$ lacking the second term.

 b) $x^4 - \frac{5}{6}x^3 + \frac{5}{12}x^2 - \frac{7}{150}x - \frac{13}{900} = 0$ with integral coefficients.

27. Find the seventh roots of – 1.