K21U 0128

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2021
(2014-2018 Admissions)
CORE COURSE IN MATHEMATICS
6B11 MAT : Numerical Methods and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Write the Newton's forward difference interpolation polynomial.
2. Give the truncation error in Euler method.
3. State the Laplacian in polar coordinates.
4. Give the one dimensional wave equation.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Find an interval which contains the root of the equation $x = e^{-x}$.
6. Perform two iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.
7. Define the finite difference operators :

   i) Forward ii) Backward and iii) Central

8. Prove that

$$\Delta\left(\frac{f_i}{g_i}\right)=\frac{g_i\Delta f_i - f_i\Delta g_i}{g_i g_i + 1}.$$

9. Construct the divided difference table for the following data :

x :	−1	0	3
f(x) :	−4	−5	16

10. Find the Lagrange interpolating polynomial that fits the data values :

x :	2.5	3.5
f(x) :	6	8

Interpolate at x = 3.

11. Using the method $\frac{1}{2h}\left[-3f_0 + 4f_1 - f_2\right]$, obtain an approximate value of $f'(-3)$ with h = 2, for the following data :

x :	− 3	− 2.5	−2	− 1
f (x) :	− 25	−14.125	−7	− 1

12. What is meant by quadrature rule and error of approximation in numerical integration ?

13. Obtain the approximate value of y(1.3) for the initial value problem $y' = -2xy^2$, y (1) = 1, using Euler method, with h = 0.1.

14. Find the approximate value of y(0.2) for the initial value problem $y = x^2 + y^2$, y(0) = 1 with h = 0.1, using Heun's method.

15. Discuss about the Runge Kutta method of solving ordinary differential equations.

16. Verify that $u = x^2 + t^2$ is a solution of the one dimension wave equation.

17. Solve the partial differential equation $u_{xy} - u_x = 0$.

18. Verify that $u(x, y) = a \ln (x^2 + y^2) + b$ is a solution of the Laplace equation and determine the values of a and b, if u satisfies the boundary conditions $u = 0$ on $x^2 + y^2 = 1$ and $u = 3$ on $x^2 + y^2 = 4$.

19. What is the solution of one dimensional wave equation, as given by Fourier series ? Deduce it for a given initial velocity.

20. Identify the type of the equation $4u_{xx} - u_{yy} = 0$ and transform it to normal form.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

21. Evaluate $\sqrt{5}$ using the equation $x^2 - 5 = 0$ by applying the fixed point iteration method.

22. Perform three iterations of the regula-falsi method to obtain the smallest positive root of $x^3 - 5x + 1 = 0$.

23. Find the second divided difference of $f(x) = \frac{1}{x}$, using the points x_0, x_1, x_2.

24. For the data

x :	0	0.2	0.4	0.6	0.8	1.0
f(x) :	7.0	0.008	5.064	4.216	3.512	3.0

Find an approximation to f (0.1) by using Newton's forward difference formula.

25. Evaluate the following integral using trapezoidal rule with n = 2

$$\int_0^1 \frac{dx}{3+2x}.$$

26. Solve by separating variables, $u_x - u_y = 0$.

27. Find the solution of the initial value problem $y' = 2y - x$, $y(0) = 1$, by performing two iterations of the Picard's method.

28. A function f(x) representing the following data has a minimum in the interval (0.5, 0.8). Find this point of minimum :

x :	0.5	0.6	0.7	0.8
f(x) :	1.3254	1.1532	0.9432	1.0514

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

29. Derive the solution of one dimensional heat equation.

30. Using Newton Raphson method, obtain the root of the equation $x^3 - 5x + 1 = 0$ correct to three decimal places. Assume $x_0 = 0$.

31. Evaluate $\int_0^1 \frac{dx}{3+2x}$ using Simpson's rule with n = 2. Compare with the exact solution.

32. Solve the initial value problem, $y' = x(y - x)$, $y(2) = 3$ in the interval [2, 2.4] using the classical Runge-Kutta fourth order method with the step size h = 0.2.

33. The following table of the function $f(x) = e^{-x}$ is given by

x :	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f(x) :	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

i) Using Gauss forward central difference formula, compute f (0.55).

ii) Using Gauss backward central difference formula, compute f (0.45).

34. Find the D'Alembert's solution of wave equation.

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