K22U 3632

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission onwards)
CORE COURSE IN MATHEMATICS
3B03 MAT : Analytic Geometry and Applications of Derivatives

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define angle of intersection of two curves.
2. The reciprocal of the curvature of the curve at any point p is called _____ at p.
3. A circle with center C, and radius ρ is called _____ at p.
4. Find the focus of the parabola $y^2 = 10x$.
5. Find critical point of $f(x) = x^2 + 2x + 3 = 0$.

PART – B

Answer **any 8** questions from 11 questions. **Each** question carries **2** marks.

6. Write the equation of a normal at any point θ to the curve $x = a(\cos\theta + \theta \sin\theta)$, $y = a(\sin\theta - \theta \cos\theta)$.
7. Find the asymptote of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.
8. Show that in the curve $r = a\theta$, the polar subnormal is constant.

P.T.O.

9. Find ρ at origin for the curve $y - x = x^2 + 2xy + y^2$.

10. Find the radius of curvature at the point (a, 0) on the curve $xy^2 = a^3 - x^3$.

11. Find the foci and asymptotes of the equation $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

12. Find the directrix of the parabola, $r = \frac{25}{10 + 10\cos\theta}$.

13. Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.

14. Determine the concavity of $y = 3 + \sin x$ on $[0, 2\pi]$.

15. Find the point of inflection of $f(x) = x^{\frac{5}{3}}$.

16. Using Maclaurin series expand tanx, up to a term containing x^3.

PART – C

Answer **any 4** questions out of 7 questions. **Each** question carries **4** marks.

17. For the cardioid $r = a(1 - \cos\theta)$, prove that $\phi = \frac{\theta}{2}$ and $p = 2a \sin^3 \frac{\theta}{2}$.

18. Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.

19. Find the asymptote of the spiral $r = \frac{a}{\theta}$.

20. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another equal cycloid.

21. Find the center, foci, vertices, asymptotes, as appropriate, of the conic sections $x^2 + 2x + 4y - 3 = 0$.

22. Find a cartesian equation for the hyperbola, centered at origin that has a focus at (3, 0) and the line $x = 1$ as corresponding directrix.

23. Let $f(x) = (x^2 - 3)e^x$. Identify the open intervals on which f is increasing and decreasing. Find the function' local and absolute extreme values.

PART – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

24. Find the equation of a tangent at any point (x, y) to the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Show that the portion of the tangent intercepted between the axes is a constant length.

25. Find the polar equation for a circle of radius a, centered at $P_0(r_0, \theta_0)$. Sketch the graph of $r = 6\cos\theta$.

26. Sketch the graph of the function $f(x) = x^4 - 4x^3 + 10$.

27. Prove that if $0 < a < b < 1$, $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.