Reg. No. : ....

Name : ....

K21P 0787

II Semester M.Sc. Degree (CBSS – Reg./Suppl. (Including Mercy Chance)/ Imp.) Examination, April 2021

(2017 Admission Onwards)

MATHEMATICS

MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Obtain the partial differential equation satisfied by all surfaces of the form $F(u, v) = 0$ where $u = u(x, y, z)$ and $v = v(x, y, z)$ are known functions of x, y and z and F is an arbitrary function of u and v having derivatives with respect to u and v.

2. Let $z = F(x, y, a)$ be a one parameter family of solutions of the first order partial differential equation $f(x, y, z, p, q) = 0$. Show that the envelope of this one parameter family, if it exists, is also a solution.

3. State maximum principle for harmonic functions. Using maximum principle prove minimum principle.

4. Let u be a solution of the Neumann problem :

$$\begin{cases} \Delta^2 u = 0 \text{ in } D \\ \dfrac{\partial u}{\partial n} = f(s) \text{ in } B. \end{cases}$$

Prove that $\int_B f(s)\, ds = 0$.

5. Convert the initial value problem :

$y'' - 5y' - 6y = 0$, $y(0) = 0$, $y'(0) = -1$ into an integral equation.

6. Find the eigenvalues of the integral equation $y(x) = \lambda \int_0^1 (2x\xi - 4x^2)\, y(\xi)\, d\xi$.

(4×4=16)

P.T.O.

PART – B

Answer **four** questions from this Part, without omitting any Unit. **Each** question carries **16** marks.

Unit – 1

7. a) Show that $(x-a)^2 + (y-b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$. By taking $b = 2a$, show that the envelope of the subfamily is $(y - 2x)^2 + 5z^2 = 5$. Also prove that $z = \pm 1$ are singular integrals.

   b) Prove that the Pfaffian differential equation :

   $(2x + y^2 + 2xz)\,dx + 2xy\,dy + x^2 dz = 0$ is integrable and find the corresponding integral.

8. a) Solve the following PDE by Jacobi's method :

   $z^2 + zu_z - u_x^2 - u_y^2 = 0.$

   b) Find a complete integral of $xpq + yq^2 - 1 = 0$ by Charpit's method.

9. a) Explain the method to find the solution of a first order semilinear equation in two variables by the method of characteristic curves.

   b) Solve $z_x + z_y = z^2$ with the initial condition $z(x, 0) = f(x)$.

Unit – 2

10. a) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$ to a canonical form.

    b) Derive d' Alembert's solution of wave equation.

11. a) Solve the following boundary value problem :

    $u_t = u_{xx},\ 0 < x < l,\ t > 0,$

    $u(0, t) = u(l, t) = 0,$

    $u(x, 0) = x(l - x),\ 0 \le x \le l.$

    b) Solve the non-homogeneous wave equation

    $u_{tt} - c^2 u_{xx} = F(x, t),\ -\infty < x < \infty,\ t > 0$

    with the homogeneous initial conditions

    $u(x, 0) = u_t(x, 0) = 0,\ -\infty < x < \infty$

    using Duhamel's principle.

12. a) What is Dirichlet problem for the upper half plane ? Using Convolution theorem prove that the solution to the Dirichlet problem for the upper half plane is

$$u(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(\xi)}{y^2 + (x - \xi)^2}\, d\xi$$

b) Using part (a) to find the solution of the Neumann problem for the upper half plane.

Unit – 3

13. a) Reduce the following boundary value problem into an integral equation :

$y'' + \lambda y = 0,\ y(0) = 0,\ y(l) = 0.$

b) Solve the integral equation $y(x) = 1 + \lambda \int_0^{\pi} \sin(x + \xi)\, y(\xi)$ by iterative method.

14. a) Determine the resolvent kernel associated with $K(x, \xi) = \cos(x + \xi)$ in the interval $[0, 2\pi]$ in the form of power series in λ. Obtain first three terms.

b) Reduce the Bessel equation

$x^2y'' + xy' + (\lambda x^2 - 1)\, y = 0$ with end conditions $y(0) = 0,\ y(1) = 0$, to a Fredholm integral equation.

15. a) Show that the integral equation :

$y(x) = x + \frac{1}{\pi} \int_0^{2\pi} \sin(x + \xi)\, y(\xi)\, d\xi$ possess no solution.

b) Solve the following integral equation by the method of successive approximations :

$y(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 x\xi y(\xi)\, d\xi.$ **(4×16=64)**