K22U 0128

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Supple./Improv.) Examination, April 2022
(2016-2018 Admissions)
CORE COURSE IN MATHEMATICS
6B11MAT : Numerical Methods and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark :

1. Write Newton's backward difference interpolation polynomial.
2. Give the one dimensional heat equation.
3. What is the order of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^3 = 0$?
4. Write down the D'Alembert's solution of wave equation.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks :

5. Verify that the smallest positive root of $x^3 - 5x + 1 = 0$ lies in the interval (0, 1).
6. Perform two iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 3x - 1 = 0$.
7. Prove that $\Delta(f_i^2) = (f_i + f_{i+1})\Delta f_i$.
8. Distinguish between linear interpolation and quadratic interpolation.
9. Using the method $f''(x_0) = \frac{1}{h^2}[f_0 - 2f_1 + f_2]$, obtain an approximate value of $f''(-1)$ with h = 1, for the following data.

x	–1	– 0.5	0	1
f(x)	2.7183	1.6487	1	0.3679

P.T.O.

10. Evaluate the following integral using trapezoidal rule with n = 2

$$\int_0^1 \frac{dx}{3+2x}.$$

11. Find the error term in the formula $f'(x_0) = \frac{1}{2h}(-3f(x_0) + 4f(x_1) - f(x_2))$.

12. Solve the IVP $y' = 2y - x$, $y(0) = 1$, by performing two iterations of Picard's method.

13. Verify that $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

14. Solve the partial differential equation $u_x + u_y = 0$, by separating variables.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks :

15. Using Newton Raphson method, find the value of $\frac{1}{18}$ upto four decimal places taking suitable initial approximation.

16. Evaluate $\sqrt{5}$ using the equation $x^2 - 5 = 0$ by applying the fixed point iteration method.

17. Find the Lagrange interpolation polynomial that fits the following data values.

x	–1	2	3	4
f(x)	–1	11	31	69

18. Find the approximate value of y(1.3) for the IVP $y' = -2xy^2$, $y(1) = 1$, using Taylor's second order method.

19. Derive Laplacian equation in polar coordinates.

20. Find the temperature in a laterally insulated bar of length L whose ends are kept temperature zero. Assume that the initial temperature is given by

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}.$$

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks :

21. The following table of the function $f(x) = e^{-x}$ is given by

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f(x)	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

i) Using Gauss forward central difference formula, compute f(0.55).

ii) Using Gauss backward central difference formula, compute f(0.45).

22. Evaluate $\int_0^2 \frac{dx}{x^2 + 2x + 10}$ using Simpson's rule with n = 2. Compare with the exact solution.

23. Solve the initial value problem, $y' = x^2 + y^2$, $y(1) = 2$ in the interval [1, 1.2] using the classical Runge-Kutta fourth order method with the step size h = 0.1.

24. Find the solution of one dimensional wave equation by using Fourier series.