K23P 0498

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2023 (2019 Admission Onwards) MATHEMATICS MAT 2C 06: Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Find $\left[Q\left(\sqrt{2}, \sqrt{3}\right) : Q\right]$.
2. Find the primitive 5^{th} root of unity in Z_{11}.
3. Distinguish between primes and irreducibles of an integral domain.
4. Is Z[i] is an integral domain ?
5. What is the order of $G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$?
6. Show that $\sqrt{1+\sqrt{5}}$ is algebraic over Q.

PART – B

Answer **4** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that every PID is a UFD. 7

   b) Prove that $Z\left[\sqrt{-5}\right]$ is an integral domain but not a UFD. 9

8. a) State and prove Kronecker's theorem. 8

   b) How could we construct a field of 4 elements ? 8

P.T.O.

9. a) State and prove Gauss's Lemma. 6

b) An ideal <p> in a PID is maximal if and only if p is irreducible. 5

c) Prove that every Euclidian domain is PID. 5

Unit – II

10. a) If α and β are constructible real numbers, then $\alpha + \beta$, $\alpha - \beta$, $\alpha\beta$ and α/β, if $\beta \neq 0$. 12

b) If E is a finite of characteristic P, then E contains exactly P^n elements for some positive n. 4

11. a) Prove that trisecting an angle is impossible. 8

b) Prove that a finite field $GF(P^n)$ of P^n elements exists for every prime power P^n. 8

12. a) State and prove Conjugation isomorphism theorem. 10

b) Define Frobenius automorphism.Also prove that $F_{\{\sigma_p\}} \cong Z_p$. 6

Unit – III

13. a) A Field E, where $F \leq E \leq K$, is a splitting field over F if only if every automorphism of $\bar{F}$ leaving F fixed maps E onto itself and thus induces an automorphism of F leaving F fixed. 12

b) Let f(x) be irreducible in F[x]. Then prove that all zeros of f(x) in $\bar{F}$ have the same multiplicity. 4

14. a) Prove that every finite field is perfect. 12

b) Find the splitting field of $x^3 - 2$ over Q. 4

15. a) State the main theorem of Galois Theory. 6

b) State and prove Primitive Element theorem. 10