K22U 1961

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Supplementary) Examination, November 2022
(2016-18 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **one** mark.

1. State Supremum property of $\mathbb{R}$.
2. Prove that a sequence in $\mathbb{R}$ can have atmost one limit.
3. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ converges.
4. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be increasing on I. If $c \in I$, prove that f is continuous at c if and only if $j_f(c) = 0$.

SECTION – B

Answer **any eight** questions, **each** question carries **two** marks.

5. Determine the set $B = \{x \in \mathbb{R} : x^2 + x > 2\}$.
6. State and prove Bernoulli's inequality.
7. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Find inf S and sup S.
8. Use the definition of the limit of a sequence to prove that $\lim\left(\frac{2n}{n+1}\right) = 2$.
9. State and prove squeeze theorem.

P.T.O.

10. Prove that $\sum_{n=0}^{\infty} r^n$ is convergent if $|r| < 1$ and divergent if $|r| \geq 1$.

11. Establish the convergence or divergence of the series whose n^{th} term is $\frac{n}{(n+1)(n+2)}$.

12. State and prove Dirichlet's test.

13. Prove that Dirichlet's function is discontinuous on $\mathbb{R}$.

14. State and prove Bolzano's intermediate value theorem.

SECTION – C

Answer **any four** questions, **each** question carries **four** marks.

15. State and prove Archimedean property.

16. State and prove nested interval property.

17. Let y_n be defined by $y_1 = 1, y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Prove that $\lim y_n = \frac{3}{2}$.

18. Prove that the p- series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$.

19. State and prove integral test.

20. State and prove uniform continuity theorem.

SECTION – D

Answer **any two** questions, **each** question carries **six** marks.

21. Prove that there exists a positive real number x such that $x^2 = 2$.

22. Prove that every contractive sequence is a Cauchy sequence.

23. a) State and prove ratio test.

    b) Establish the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

24. State and prove location of roots theorem.