K24U 4025

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree (C.B.C.S.S. – OBE-Supplementary/ Improvement) Examination, November 2024
(2019 to 2023 Admission)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT-BCA : Mathematics for BCA – I

Time : 3 Hours   Max. Marks : 40

SECTION – A

Questions **1-5**, answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Show that $\frac{d}{dx}(\cos^{-1}x + \sin^{-1}x) = 0$.
2. Find the derivative of $\sqrt{\frac{e^x + e^{-x}}{2}}$.
3. Write the dual of the following statement : $a' * (a + b) = a' * b$.
4. Find the rank of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$.
5. Show that A^{-1} is orthogonal if A is orthogonal.

SECTION – B

Questions **6-15**, answer **any seven** questions. **Each** question carries **two** marks. **(7×2=14)**

6. Find the derivative of $\log(x + \sqrt{x^2 + 1})$.
7. Given that $y = \sin(\log x)$. Prove that $x^2y_2 + xy_1^2 + y = 0$.
8. Find the n^{th} derivative of $\cos(x/2)$.
9. Given that $x = t^2 + 1$, $y = 2t - 1$. Find $\frac{d^2y}{dx^2}$.

P.T.O.

10. Prove that in a Boolean Algebra B, $x'' = x$ for all $x \in B$.

11. Give an example for a Boolean Algebra with two elements.

12. Find the normal form of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

13. Show that the matrix $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is orthogonal.

14. Find the value of λ such that the rank of the matrix $\begin{pmatrix} \lambda & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & \lambda \end{pmatrix}$ is 2.

15. Does the set of equations $2x + y + z = 0$, $x - y + z = -1$, $3x + 2z = -1$ are consistent ? Justify your answer.

SECTION – C

Questions **16-22**, answer **any four** questions. **Each** question carries **three** marks. **(4×3=12)**

16. Derive the derivative of $\operatorname{cosec}^{-1}x$.

17. Find $\frac{dy}{dx}$, if $y = \frac{\sqrt{\sin x + \sin 2x + \sin 3x}}{\cos x + \cos 2x + \cos 3x}$.

18. Given that $x^2 + y^2 + 4xy = 0$. Prove that $\frac{dy}{dx} = \frac{-(x + 2y)}{(2x + y)}$.

19. Find the n^{th} derivative of $e^x \cos x$.

20. State and prove the Absorption Laws in a Boolean Algebra B.

21. Solve the system of equations $x + y + z = 1$, $14x + 7y + 7z = 4$, $7x + 14y - 7z = 1$ using Crammer's rule.

22. Show that the vectors $x_1 = (-1, 2, 3, 0)$, $x_2 = (2, 0, 3, 0)$, $x_3 = (1, 0, 0, -1)$ are linearly independent.

SECTION – D

Questions **23-26**, answer **any two** questions. **Each** question carries **five** marks. **(2×5=10)**

23. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

24. Find $\frac{dy}{dx}$ for the following :

    a) $y = (\log x)^x + x^x$ b) $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

25. State and prove the De Morgan's Laws in a Boolean Algebra B.

26. Investigate the values of μ and λ so that the equations $2x + 3y + 5z = 9$, $7x + y - 2z = 8$, $2x + y + \lambda z = \mu$ have

    i) no solution ii) a unique solution iii) an infinite number of solutions.