K21P 0785

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Suppl. (Including Mercy Chance)/Imp.) Examination, April 2021
(2017 Admission Onwards)
MATHEMATICS
MAT2C08 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Is a bounded metric space necessarily totally bounded ? Justify your answer.
2. Show that the real line with the usual topology is locally compact but not compact.
3. Give an example of a T_0-space that is not a T_1-space.
4. Let $X = \{1, 2, 3\}$ and $\tau = \{\phi, \{1\}, \{1, 3\}, X\}$. Determine whether (X, τ) is a regular space.
5. For each $n \in \mathbb{N}$, let (X_n, d_n) be a metric space. Assume $d_n(x_n, y_n) \leq 1$ for each $n \in \mathbb{N}$ and all $x_n, y_n \in X_n$. Show that there is a metric on $X = \prod_{n\in\mathbb{N}} X_n$.
6. Show that every contractible space is pathwise connected. (4×4=16)

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) When is a metric space said to be totally bounded ? Prove that every totally bounded metric space is bounded.

   b) Prove that a metric space is compact if and only if it is complete and totally bounded.

8. a) Let (X, τ) be a compact space, (Y, U) be a Hausdorff space and let $f : X \to Y$ be a continuous function. Prove that f is a closed mapping.

   b) Let (X, τ) be a topological space and let $\mathcal{B}$ be a basis for τ. Prove that (X, τ) is compact if and only if every cover of X by members of $\mathcal{B}$ has a finite subcover.

   c) Prove that the product of two compact spaces is compact.

P.T.O.

9. a) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.

   b) Let $\{(X_\alpha, \tau_\alpha) : \alpha \in \wedge\}$ be a collection of spaces, and let τ be the product topology on $X = \prod_{\alpha \in \wedge} X_\alpha$. Prove that (X, τ) is locally compact if and only if for each $\alpha \in \wedge$, (X_α, τ_α) is locally compact and for all but a finite number of $\alpha \in \wedge$, (X_α, τ_α) is compact.

Unit – II

10. a) Prove that a T_1-space (X, τ) is regular if and only if for each member p of X and each neighborhood U of p there is a neighborhood V of p such that $\overline{V} \subseteq U$.

    b) Prove that every subspace of a regular space is regular.

    c) Define a complete regular space and prove that every completely regular space is regular.

11. a) Let $(X, \leq)$ be a well ordered set, and let τ denote the order topology on X. Prove that (X, τ) is a normal space.

    b) Prove that a T_1 – space (X, τ) is completely normal if and only if every subspace of it is normal.

12. a) Prove that every second countable space is Lindelof. Show by an example that a second countable space need not be Lindelof.

    b) Prove that every regular Lindelof space is normal.

Unit – III

13. a) State and prove Urysohn's lemma.

    b) Deduce that every normal space is completely regular.

14. a) State and prove Tychonoff theorem.

    b) Let (X, τ) be a T_1-space. Prove that (X, τ) is regular and second countable if and only if it is a separable metric space.

15. a) Let (X, τ) be a topological space and let $x_0 \in X$. Also let $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \Omega(X, x_0)$ and suppose $\alpha_1 =_p \alpha_2$ and $\beta_1 =_p \beta_2$. Prove that $\alpha_1 * \beta_1 = \alpha_2 * \beta_2$.

    b) Let (X, τ) be a topological space, let $x_0 \in X$ and let $\alpha, \beta, \gamma \in \pi_1(X, x_0)$. Prove that $([\alpha] \circ [\beta]) \circ [\gamma] = [\alpha] \circ ([\beta] \circ [\gamma])$. **(4×16=64)**

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