K22P 1411

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT3C14 – Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. (4×4=16)

1. Let B be the uniform closure of an algebra A of bounded functions. Then prove that B is a uniformly closed algebra.

2. Give an example of a functions with f_n converges to f, but f_n' does not converges to f'. Justify your answer.

3. Define orthogonal system of functions. Give example with justification.

4. Prove that $\lim_{x \to +\infty} x^{-\alpha} \log x = 0$.

5. Prove that the existence of all partial derivatives does not imply the differentiability.

6. Explain directional derivative of f at x in the direction of a unit vector u and continuously differentiable functions.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks. (4×16=64)

Unit – I

7. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n$, (n = 1, 2, 3, ...). Then Prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{t \to \infty} A_n$.

P.T.O.

b) Suppose K is compact, and

i) $\{f_n\}$ is a sequence of continuous functions on K,

ii) $\{f_n\}$ converges pointwise to a continuous function f on K,

iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3 ... Then prove that $f_n \to f$ uniformly on K.

8. a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

b) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

9. Let A be an algebra of real continuous functions on a compact set K. If A separates points on K and if A varnishes at no point of K, then prove that the uniform closure B of A consists of all real continuous functions on K.

Unit – II

10. a) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $(|x| < R)$. Then prove that the series $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R + \in, R - \in]$, no matter which $\in > 0$ is chosen. Also prove that the function f is continuous and differentiable in (– R, R) and $f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1}$, $|x| < R$.

b) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $(|x| < R)$. Then prove that f has derivatives of all orders in (– R, R) and derive the formulas.

11. State and prove Parseval's Theorem.

12. a) Define Gamma Function. Prove that $\log\Gamma$ is convex on $(0, \infty)$.

b) State and prove Stiriling's Formula.

Unit – III

13. a) Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that dim $X \leq r$.

b) Suppose X is a vector space, and dim X = n. Prove that

i) A set E of n vectors in X spans X if and only if E is independent.

ii) X has a basis and every basis consists of n vectors.

iii) If $1 \leq r \leq n$ and $\{y_1, y_2, ..., y_r\}$ is an independent set in X then X has a basis containing $\{y_1, y_2, ..., y_r\}$.

14. a) Suppose f maps an open set $E \subset R^n$ into R^m. Then prove that $f \in C(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq n$.

b) Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E and there is a real number M such that $||f'(x)|| \leq M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \leq M|b - a|$ for all $a \in E$, $b \in E$.

15. State and prove implicit function theorem.
