



K21U 3609

Reg. No. :

Name :



II Semester B.Sc. Degree (CBCSS-Supple.) Examination, April 2021
(2014-2016 Admission)
CORE COURSE IN MATHEMATICS
2B02MAT : Integral Calculus

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry 1 mark each.

1. State the shift property of definite integrals.
2. Evaluate $B(4, 5)$.
3. Write the polar coordinate formula for the length of a curve $r = f(\theta)$, $\alpha \leq \theta \leq \beta$.
4. Evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2}} dy dx$.

SECTION – B

Answer **any 8** questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Show that the value of $\int_0^1 \sqrt{1+\cos x} dx$ cannot possibly be 2.
6. Evaluate $\int_{-1}^1 3x^2 \sqrt{x^3+1} dx$.
7. Find the derivative of y w.r.t. x where $y = \sinh^{-1}(\tan x)$.
8. Show that $\int_0^{\infty} \frac{\cos x}{x^2+1} dx$ converges.

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9. Show that $B(u, v) = B(v, u)$, where $B(u, v)$ is the beta function.
10. Describe the points of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which cuts the coordinate axes $(\pm a, 0, 0)$, $(0, \pm b, 0)$ and $(0, 0, \pm c)$. Also write the equation of the section of the surface by the plane $z = k$, which is parallel to XY plane.
11. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\frac{\pi}{4}$.
12. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$ about y-axis.
13. Find the average of $f(x, y) = x \cos y$ over the rectangle R , $0 \leq x \leq \pi$, $0 \leq y \leq 1$.
14. Evaluate $\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{2-r^2}} 3 \, dz \, r \, dr \, d\theta$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find the area of the region between the curve $y = 4 - x^2$, $0 \leq x \leq 3$ and the x-axis.
16. Find $\int_0^{\infty} x^6 \cdot e^{-2x} \, dx$.
17. Find the surface area of the solid generated by revolution of the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about the x-axis.
18. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.
19. Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.
20. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y} (y-2x)^2 \, dy \, dx$.



SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. i) Find the derivative of $\ln x + \sqrt{1-x^2} \operatorname{sech}^{-1} x$.

ii) Find $\int \tanh^{-1} x \, dx$.

22. Prove that $B(u, v) = \frac{\Gamma(u) \Gamma(v)}{\Gamma(u+v)}$, $u, v > 0$.

23. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.

24. Find the centroid of the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$.
