K20U 1533

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B06MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions **each** question carries **1** mark :

1. On $\mathbb{Z}^+$, define $*$ by letting $a * b = a^b$. Find $(2 * 2) * 3$.
2. What is the order of dihedral group D_4 ?
3. Let G be a group and let $\phi : G \to G$ by $\phi(g) = g^{-1}$. Is ϕ a homomorphism ?
4. Find the number of zero divisors of the ring $\mathbb{Z}_6$. (4×1=4)

SECTION – B

Answer **any eight** questions **each** question carries **2** marks :

5. State and prove the left cancellation law of groups.
6. Find the remainder when – 61 is divided by 7.
7. Compute $\tau\sigma^2$, where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$.
8. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$ into product cycles and transpositions.
9. Write all the left cosets of $4\mathbb{Z}$ of $\mathbb{Z}$.

P.T.O.

10. Prove that a group homomorphism $\phi : G \to G'$ is one-to-one map if and only if Ker $(\phi) = \{e\}$.

11. Find the order of $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.

12. Let R be a ring with additive identity 0. Show that $(-a)(-b) = ab$, for any $a, b \in R$.

13. Define characteristic of a ring and give an example of a ring with characteristic 59.

14. Find all solutions of $2x \equiv 6 \pmod 4$. **(8×2=16)**

SECTION – C

Answer **any four** questions **each** question carries **4** marks :

15. Prove that subgroup of a cyclic group is cyclic.

16. Show that every permutation on a finite set is a product of disjoint cycles.

17. State and prove Lagrange's theorem.

18. Show that a subgroup H of G is a normal subgroup if and only if $ghg^{-1} \in H$, for all $g \in G$ and $h \in H$.

19. Prove that every field is an integral domain.

20. If $a \in \mathbb{Z}$ and p is a prime not dividing a. Show that p divides $a^{p-1} - 1$. **(4×4=16)**

SECTION – D

Answer **any two** questions **each** question carries **6** marks :

21. Let G be a cyclic group with n elements and generated by a. Let $b \in G$ and let $b = a^s$. Prove that b generates a cyclic subgroup H of G containing n/d elements.

22. Prove that every group is isomorphic to a group of permutation.

23. State and prove the fundamental homomorphism theorem.

24. Let m be a positive integer and let $a, b \in \mathbb{Z}_m$. Let d be the gcd of a and b. Prove that the equation $ax = b$ has a solution in $\mathbb{Z}_m$ if and only if d divides b. **(2×6=12)**