



K20U 1534

Reg. No. : .....

Name : .....



V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination,  
November 2020

(2014 Admn. Onwards)

CORE COURSE IN MATHEMATICS

5B07 MAT : Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **all 4** questions :

(4×1=4)

1. Why the differential equation  $y' + x^2y = \frac{1}{y}$  is linear ? Justify.
2. Find the Wronskian of  $y_1 = e^{2t}$ ,  $y_2 = e^{-3t}$ .
3. Define Unit step function.
4. Show that the sum of two even functions is even.

PART – B

Answer **any 8** questions :

(8×2=16)

5. Solve the differential equation  $y' = (1 + x)(1 + y^2)$ .
6. Check whether the equation  $\cos(x + y)dx + (3y^2 + 2y + \cos(x + y))dy = 0$  is exact.
7. Solve the differential equation  $y'' - 6y' + 9y = 0$ .
8. Find a particular solution of  $y'' - 2y' - 3y = 3e^{2t}$ .
9. Find the general solution of  $(D^2 + 3I)y = 0$ , where D is the differential operator.

P.T.O.



10. Find the Laplace transform of the function  $f(t) = \begin{cases} 2, & 0 < t < \pi \\ 0, & \pi < t < \infty \end{cases}$ .
11. Find  $L(te^{-t} \sin 3t)$ .
12. Find the inverse Laplace transform of the function  $\frac{1}{s(s^2 + \omega^2)}$ .
13. Sketch the graph of the function  $f(x) = |x|$  if  $-2 \leq x \leq 2$  and  $f(x + 4) = f(x)$ .
14. If  $f$  and  $g$  are periodic functions with same period  $T$ , show that any linear combinations of  $f$  and  $g$  is also  $T$ -periodic.

## PART – C

Answer any 4 questions :

(4×4=16)

15. Solve the differential equation  $xy' + y = xy^3$ .
16. Given that  $Y_1$  and  $Y_2$  are solutions of the equation  $y'' + p(t)y' + q(t)y = 0$ . Prove that for any two constants  $c_1$  and  $c_2$ , the linear combination  $c_1Y_1 + c_2Y_2$  is also a solution for the differential equation.
17. Find the general solution of  $t^2y'' - 4ty' + 6y = 0$ ,  $t > 0$ .
18. Assuming the required conditions, prove that  $L[f'(t)] = sL[f(t)] - f(0)$ .
19. Find the Fourier cosine series expansion of  $f(x) = 2 - x$  when  $0 \leq x \leq 2$  with period 4.
20. Find the Fourier integral representation of the function  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ .

## PART – D

Answer any 2 questions :

(2×6=12)

21. Find the orthogonal trajectories of the families of curves  $\frac{1}{2}x^2 + y^2 = c$ .
22. By method of variation of parameters, solve the differential equation,  $y'' - 5y' + 6y = 2e^t$ .
23. State and prove convolution theorem for Laplace transform.
24. Find the Fourier series of the function  $f(x) = x + \pi$  if  $-\pi < x < \pi$  and  $f(x + 2\pi) = f(x)$ .