



K20U 3184

Reg. No. : .....

Name : .....

I Semester B.Sc. Degree (CBCSS – Supplementary)

Examination, November 2020

(2017-2018 Admissions)

CORE COURSE IN MATHEMATICS

1B01 MAT : Differential Calculus

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each :

1. Find  $\lim_{t \rightarrow 6} 8(t - 5)(t - 7)$ .
2. Find  $\frac{d}{dx} \sinh \frac{x}{3}$ .
3. Find the Cartesian equation for  $r^2 = 4r \cos \theta$ .
4. Find an equation for the cylinder  $x^2 + (y - 3)^2 = 9$  in cylindrical coordinates.

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each :

5. Evaluate  $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$ .
6. For what value of a is  $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$  continuous at every  $x = 2$ .
7. Express  $\sinh^{-1} \left( \frac{-5}{12} \right)$  in terms of natural logarithms.

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8. Find the Cartesian equation for  $\rho = 5 \cos \phi$ .
9. Find  $\frac{dS}{d\theta}$  for the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ .
10. Find the radius of curvature of the catenary  $y = c \cosh\left(\frac{x}{c}\right)$  at  $(0, c)$ .
11. Verify Lagrange's mean value theorem for  $f(x) = \log_e x$  in  $[1, e]$  and find appropriate value for  $c$ .
12. Find  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right)$ .
13. Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ .
14. Find the value of  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  at the point  $(4, -5)$  if  $f(x, y) = x^2 + 3xy + y - 1$ .

## SECTION - C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4 marks each** :

15. If  $ax^2 + 2hxy + by^2 = 1$ , prove that  $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}$ .
16. Find the radius of curvature at the point  $(r, \theta)$  on the curve  $r^n = a^n \cos n\theta$ .
17. Using Maclaurin's series, expand  $\sin x$ .
18. Verify Rolle's theorem for  $f(x) = (x + 2)^3(x - 3)^4$  in  $(-2, 3)$ .
19. Express  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial s}$  in terms of  $r$  and  $s$ , if  $w = x^2 + y^2$ ,  $x = r - s$ ,  $y = r + s$ .
20. Verify Euler's theorem for  $z = ax^2 + 2hxy + by^2$ .



## SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6 marks each** :

21. If  $y^{\frac{1}{n}} + y^{\frac{1}{m}} = 2x$  prove that  $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ .

22. Find the evolute of the curve  $x = a\cos^3\theta$ ,  $y = a\sin^3\theta$ .

23. Find the volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius  $a$ .

24. If  $u = \frac{x^2y^2}{x^2 + y^2}$ , show that

i)  $x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial u}{\partial x}$

ii)  $x \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y}$

iii)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u$ .

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