K25P 2016

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Supplementary) Examination, April 2025
(2021 and 2022 Admissions)
MATHEMATICS
MAT2C06 : Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define UFD and give an example of a UFD.
2. Prove that every Euclidean domain is a PID.
3. Find the degree and a basis for $\mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{18}\right)$ over $\mathbb{Q}$.
4. Define primitive n^{th} root of unity. Find the number of 18^{th} roots of unity in GF(19).
5. Find the degree over $\mathbb{Q}$ of the splitting field over $\mathbb{Q}$ of $(x^2 - 2)(x^2 - 3)$ in $\mathbb{Q}[x]$.
6. If E is a finite extension of F, show that {E : F} divides [E : F]. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) If D is a UFD, prove that product of two primitive polynomials in D[x] is again primitive. 6

   b) If D is a UFD, prove that D[x] is a UFD. 10

8. a) State and prove Euclidean Algorithm. 12

   b) Compute the gcd of 22471 and 3266. 4

P.T.O.

9. a) Let F be a field and let f(x) be a non-constant polynomial in F[x]. Prove that there exist an extension field E of F and an $\alpha \in E$ such that $f(\alpha) = 0$. 11

b) Prove that there exists a field of 27 elements. 5

Unit – II

10. Prove that every field F has an algebraic closure $\overline{F}$.

11. a) State and prove Fundamental theorem of algebra. 5

b) Prove that $\mathbb{Q}\left(\sqrt{3}+\sqrt{7}\right)=\mathbb{Q}\left(\sqrt{3},\sqrt{7}\right)$. 7

c) Let E be a finite extension of degree n over a finite field F. If F has q elements, prove that E has q^n elements. 4

12. a) Prove that doubling the cube is impossible. 6

b) Show that the set of all automorphisms of a field E is a group under function composition. 4

c) Let F be a finite field of characteristic p. Prove that $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$ for $a \in F$ is an automorphism and $F_{\{\sigma_p\}} = \mathbb{Z}_p$. 6

Unit – III

13. State and prove isomorphism extension theorem.

14. a) If $E \leq \overline{F}$ is a splitting field over F, prove that every irreducible polynomial in F[x] having zero in E splits in E. 5

b) If E is a finite extension of F, prove that E is separable over F if and only if each α in E is separable over F. 6

c) What is the order of $G\left(\mathbb{Q}\left(\sqrt[3]{2}, i\sqrt{3}\right)/\mathbb{Q}\right)$? 5

15. a) Let E be a finite field of order p^n.

i) Show that the Frobenius automorphism, σ_p has order n. 4

ii) Deduce from (i) that G(E/F) is cyclic of order n with generator σ_p. 3

b) Let K be a finite normal extension of a field F, with Galois group G(K/F). For a field E, where $F \leq E \leq K$, let $\lambda(E)$ be the subgroup of G(K/F) leaving E fixed. Prove that λ is a one to one map of the set of all such intermediate fields E on to the set of all subgroups of G(K/F). 9

(4×16=64)