K23P 1409

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, October 2023 (2020 Admission Onwards) MATHEMATICS MAT3C12 : Functional Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. State and prove Riesz lemma.
2. Show that c_{00} cannot be a Banach space with respect to any norm.
3. If a closed map F is bijective, then show that its inverse F^{-1} is also closed.
4. State open mapping theorem.
5. Let X be an inner product space and $x \in X$. Prove that $\langle x, y \rangle = 0$ for all $y \in X$ if and only if $x = 0$.
6. Let E be an orthogonal subset of an inner product space X and $0 \notin E$. Show that E is linearly independent.

PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Define a normed space and draw the sets $\left\{x \in \mathbb{R}^2 ; \| x \|_p = 1\right\}$ for p = 1, 2 and ∞.

   b) If X is a finite dimensional normed space then show that every closed and bounded subset of X is compact.

P.T.O.

8. a) Show that every linear map from a finite dimensional normed space is continuous.

   b) Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that R(F) of F is finite dimensional. Show that F is continuous if and only if the zero space Z(F) is closed in X.

9. a) State and prove Hahn-Banach separation theorem.

   b) If X is a normed space and X' is strictly convex then show that for every subspace Y of X and every $g \in Y'$, there is a unique Hahn-Banach extension of g to X.

Unit – II

10. a) State and prove Uniform Boundedness Principle.

    b) Give the geometric interpretation of Uniform Boundedness Principle.

11. State and prove Closed Graph Theorem.

12. a) State and prove Bounded Inverse Theorem.

    b) Let X be a Banach space in the norm $\| \ \|$. Show that there is a norm $\| \ \|'$ on X which is comparable to the norm $\| \ \|$, but in which X is not complete.

Unit – III

13. a) State and prove Gram-Schmidt orthonormalization process.

    b) State and prove Riesz-Fischer theorem.

14. a) If H is a non-zero separable Hilbert space over K then show that H has a countable orthonormal basis.

    b) If E is a convex subset of an inner product space X, then show that there exists at most one best approximation from E to X.

15. a) State and prove Riesz representation theorem.

    b) Let H be a Hilbert space and for $f \in H'$, let y_f be the representer of f in H. Show that the map $T : H \to H'$ given by $T(f) = y_f$ is a surjective conjugate-linear isometry.

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