K23P 1412

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.)
Examination, October 2023
(2020 Admission Onwards)
MATHEMATICS
MAT3E01 : Graph Theory

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks. **(4×4=16)**

1. Define independent set of a graph G. Prove that a set $S \subset V$ is an independent set of G if and only if $S - V$ is a covering of G.

2. If $\delta > 0$, then prove that $\alpha' + \beta' = v$ where α' and β' where $\alpha'(G)$ and $\beta'(G)$ are the edge independence number and edge covering number of G respectively.

3. Show that the Peterson graph is 4–edge chromatic.

4. Prove that a graph G is embeddable in the plane if and only if it is embeddable on the sphere.

5. Prove that if G is a k-regular bipartite graph with $k > 0$, then G has a perfect matching.

6. Prove that a simple graph G is connected if and only if, given any pair of distinct vertices u and v of G, there are at least n internally disjoint paths from u to v.

P.T.O.

PART – B

Answer **any 4** questions without omitting any **unit**. **Each** question carries **16** marks.

UNIT – I

7. a) State and prove Ramsey's theorem.

   b) Let $(S_1, S_2, ..., S_n)$ be any partition of the set of integers 1, 2, ..., r_n. Then, prove that for some i, S_i contains three integers x, y and z satisfying the equation $x + y = z$.

8. a) If $\{x_1, x_2, ..., x_n\}$ is a set of diameter 1 in the plane, then prove that the maximum possible number of pairs of points at distance greater than $1/\sqrt{2}$ is $[n^2/3]$. Also prove that for each n, there is a set $\{x_1, x_2, ..., x_n\}$ of diameter 1 with exactly $[n^2/3]$ pairs of points at distance greater than $1/\sqrt{2}$.

   b) If G is simple and contains no K_{m+1}, then prove that $\varepsilon(G) \leq \varepsilon(T_{m,v})$. Also prove that $\varepsilon(G) = \varepsilon(T_{m,v})$ only if $G = T_{m,v}$.

9. a) If G is k-critical, then prove that $\delta \geq k - 1$.

   b) Show that every k-chromatic graph has at least k vertices of degree at least $k - 1$.

   c) Prove that in a critical graph, no vertex is a clique.

UNIT – II

10. a) If two bridges overlap, then show that either they are skew or else they are equivalent 3-bridges.

    b) Show that $K_{3,3}$ is non-planar.

    c) Prove that an inner bridge that avoids every outer bridge is transferable.

11. a) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colors are represented at each vertex of degree at least two.

    b) If G is bipartite, then prove that $X' = \Delta$.

12. a) Let M and N be disjoint matchings of G with $|M| > |N|$. Prove that there are disjoint matchings M' and N' of G such that $|M'| = |M| - 1$, $|N'| = |N| + 1$ and $M' \cup N' = M \cup N$.

b) Show that a graph is planar if and only if each of its blocks is planar.

UNIT – III

13. a) Prove that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

b) In a bipartite graph, show that the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

14. Prove that G has a perfect matching if and only if $o(G - S) \le |S|$ for all $S \subset V$.

15. State and prove Menger's theorem. **(4×16=64)**