K20U 3320

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree CBCSS (OBE) – Reg./Supple./Improve. Examination, November 2020
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
1B01 MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question is of **1** mark.

1. Find gof(2) if $f : R \to R$ and $g : R \to R$ are given by $f(x) = 3x - 1$; $g(x) = x^2 - 2$.
2. Find the limit $\lim_{x \to 0} \frac{\tan x}{x}$.
3. If $z = xyf(x/y)$, find the value of 'n' such that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = (n-1)z$.
4. Find the first order partial derivatives of $z = e^{-x+y}$.
5. Is the relation R = {(1, 1), (1, 2), (2, 2)} a partial order on {1, 2} ? Justify your answer.

PART – B

Answer **any 8** questions. **Each** question is of **2** marks.

6. Check if the function $f : R \to R$ given by $f(x) = \frac{3x+1}{2}$ is one-to-one and onto.
7. Check if the relation f = {(2, 3), (1, 4), (2, 1), (3, 2), (4, 3)} from A = {1, 2, 3, 4} to itself is a function or not.
8. On the set A = {1, 2, 3, 4, 5, 6}, consider the relation

   R = {(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)}. Find the partitions of A induced by R.

P.T.O.

9. Define equivalence relation and check if R = {(1, 1), (1, 2), (2, 2), (3, 3), (1, 3)} on the set A = {1, 2, 3} is an equivalence relation or not.

10. Give an example of a relation on A = {1, 2, 3, 4, 5} which is both an equivalence relation and a partial order on it. Justify.

11. Is the function $f(x,y) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 2, & \text{if } x = 1 \end{cases}$ continuous at x = 1 ? Justify your answer.

12. Find the limit $\lim_{x \to 0} \frac{x}{|x|}$, if exists. Justify your answer.

13. If a > 0, $a \leq f(x) \leq a + x$, $a - x \leq g(x) \leq a$ and both the limits $\lim_{x \to 0} f(x)$, $\lim_{x \to 0} g(x)$ exist, find $\lim_{x \to 0} \frac{f(x)}{g(x)}$.

14. If $z = u^2 + v^2$ and $u = at^2$, $v = 2at$, find $\frac{dz}{dt}$ using chain rule.

15. Find $\frac{dy}{dx}$ if $x^y = y^x$.

16. Find a root of $xe^x - 2 = 0$ using bisection method.

PART – C

Answer **any 4** questions. **Each** question is of **4** marks.

17. On the set of all natural numbers N, define a relation R by (a, b) ∈ R if '6 divides a – b'. Show that the relation is an equivalence relation and write down the collection of all equivalence classes.

18. Show that for a function f, $\lim_{x \to c} |f(x)| = 0$ implies $\lim_{x \to c} f(x) = 0$. Is it true that $\lim_{x \to c} |f(x)| = 1$ implies $\lim_{x \to c} f(x) = 1$? Justify.

19. Evaluate the following limits :

i) $\lim_{x \to 3} \frac{x^3 - 27}{x^4 - 81}$

ii) $\lim_{h \to 0} \frac{\sqrt{5h+4} - 2}{h}$.

20. For $f(x,y) = \frac{3x-y}{2x+y}$, find the limits $\lim_{x\to 0}(\lim_{y\to 0} f(x,y))$ and $\lim_{y\to 0}(\lim_{x\to 0} f(x,y))$. Is this function continuous at (0, 0) ? Justify your claim.

21. If $u = e^x \cos(y)$, $v = e^x \sin(y)$ and f(x, y) is any function of x and y, then show that

i) $\frac{\partial f}{\partial x} = u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v}$

ii) $\frac{\partial f}{\partial y} = -v\frac{\partial f}{\partial u} + u\frac{\partial f}{\partial v}$.

22. Show that if $y = f(x + at) + g(x - at)$ with f and g twice differentiable, then $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$.

23. Determine the root of $x^4 + x^3 - 7x^2 - x + 5 = 0$, which lies in between 2 and 3 using Regula-falsi method, correct to three decimal places.

PART – D

Answer **any 2** questions. **Each** question is of **6** marks.

24. Let $f : A \to B$, $g : B \to C$ be two functions. Prove that

i) If both f and g are one-to-one, then $g \circ f$ is also one-to-one.

ii) If both f and g are onto, then $g \circ f$ is also onto.

iii) If $g \circ f$ is one-to-one, then f must be one-to-one.

25. If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. Hence find the value of y_n when x = 0.

26. If $u = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$.

27. Use Newton-Raphson method to find the root of $x^4 - x - 10 = 0$ which is near to 2, correct to three decimal places.