M 26751

Reg. No. : ....

Name : ....

I Semester M.C.A. (Reg./Sup./Imp.) Degree Examination, February 2015
(2013 and Earlier Admn.)
MCA C1.3 : DISCRETE MATHEMATICS

Time : 3 Hours | Max. Marks : 80

***Instruction** : Answer **any five** questions.*

1. a) Show that $(\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)) \Leftrightarrow r$. 5

   b) Obtain the principle disjunctive normal form of $(\sim p \vee \sim q) \rightarrow (\sim p \wedge r)$. 5

   c) Define tautology. Show that 6

   $((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q$ is a tautology.

2. a) If A, B, C are sets then show that : 8

   i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

   b) Define an equivalence relation. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) \mid x - y$ is divisible by 3$\}$. Show that R is an equivalence relation. Draw the graph of R and write its matrix. 8

3. a) Define the converse, inverse and contrapositive of a conditional statement. State the converse, inverse and contrapositive to the following statement. "If triangle ABC is a right angled triangle, then $|AB|^2 + |BC|^2 = |AC|^2$". 6

   b) Find the selection of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$ with the initial conditions $a_0 = 1$ and $a_1 = 8$. 4

   c) Use mathematical induction to show that $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$. 6

4. a) Let $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ be relation matrices of the relations R and S respectively. Show that $M_{\overline{R \circ S}} = M_{\bar{S} \circ \bar{R}}$.

b) Define one-one and onto function. Give an example each.

c) State Pigeonhole principle. How many people among 2,00,000 people are born at the same time (hour, minute, seconds) ?

5. a) Let $f : A \to B$ and $g : B \to C$ be both one-one and onto functions. Then show that $g \circ f : A \to C$ is also one-one and onto.

b) Show that :

i) $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$

ii) $r.\,{}^{n}C_r = n\,{}^{(n-1)}C_{r-1}$.

c) Let A = {1, 2, 3, 4, 12} . Consider the partial order of divisibility on A. That is, if a and b are in A, $a \leq b$ if and only if a|b. Draw the Hasse diagram of (A, $\leq$). 4

6. a) Show that the set G = {0, 1, 2, 3, 4} is an abelian group with respect to addition modulo 5. 8

b) State and prove Lagrange's theorem. 4

c) Prove that the intersection of any two subgroups of G is again a subgroup of G. 4

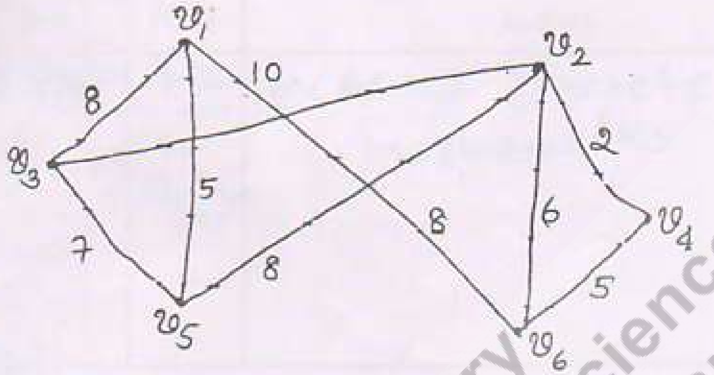
7. a) State and prove the addition theorem of probability. 5

b) A problem in Mathematics is given to three students whose chances of solving the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved ? 5

c) Explain path, reachability and connectedness. 6

a) Show that a tree with n vertices has exactly (n – 1) edges. 6

b) Using Kruskal's algorithm find the minimal spanning tree of the graph shown below. 5



c) In a complete graph with n vertices prove that there are (n – 1)/2 edge-disjoint Hamiltonian cycles, if n is an odd number ≥ 3. 5