K22U 3419

Reg. No. : .....

Name : .....

I Semester B.Sc. Degree (C.B.C.S.S.– O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2022

(2019 Admission Onwards)

CORE COURSE IN MATHEMATICS

1B01 MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours Max. Marks : 48

PART – A

Answer **four** questions from this Part. **Each** question carries **one** mark.

1. Define a relation on set of integers.
2. Find the n^{th} derivative of $x^3 + 5x^2 + 3$.
3. Find the domain of log x.
4. State Euler's theorem on homogeneous functions.
5. Let $w = x^2y + 2y$. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

PART – B

Answer **any eight** questions from this Part. **Each** question carries **two** marks.

6. Define reflexive relation on a set and give an example.
7. Show that the relation $\leq$, is a partial relation on set of all real numbers.
8. Give an example of a function which is one-one, but not on-to.
9. Give an example of algebraic and transcendental equation.
10. Find $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ 1 & \text{if } x < 0 \end{cases}$

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11. Show that $\lim_{\theta \to 0} \sin\theta = 0$.

12. For what values of a is $f(x) = \begin{cases} a^2x - 2a, \text{ if } x \geq 2 \\ 12 \text{ if } x \leq 2 \end{cases}$ continuous at every x ?

13. Describe the domain of the function $f(x, y) = \sqrt{y - x^2}$.

14. Find the values of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ at the point (4, –5) if $f(x, y) = x^2 + 3xy + y - 1$.

15. Let $f(x, y) = 2x + 3y - 4$. Find the slope of the line tangent to this surface at the point (2, –1) and lying in the plane $x = 2$.

16. Find $\frac{dy}{dx}$ using implicit differentiation, if $y^2 - x^2 - \sin xy = 0$.

PART – C

Answer **any four** questions from this Part. **Each** question carries **four** marks.

17. Show that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

18. Let A = {a, b} and B = {1, 2, 3}.
    i) Find number of functions from A to B
    ii) Find number of functions from B to A.

19. Determine the maximum number of positive and negative roots and intervals of length one unit in which the real roots lie for the following equation $8x^3 - 12x^2 - 2x + 3 = 0$.

20. If $ax^2 + 2hxy + by^2 = 1$, then show that $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}$.

21. Evaluate $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

22. Describe the level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

23. State mixed partial theorem. Verify it for the function $w = x \sin y + y \sin x + xy$.

PART – D

Answer **any two** questions from this Part. **Each** question carries **six** marks.

24. i) Let $f : R \to R$, defined by $f(x) = 2x - 3$, find the formula for f^{-1}.

    ii) Consider the function $f : A \to B$, $g : B \to C$, then prove that if $g \circ f$ is one-one, then f is one-one.

25. Find the root correct to two decimal places of the equation $f(x) = xe^x - \cos x = 0$, using the method of false position.

26. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then show that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

27. i) Find $\lim_{(x,y) \to (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

    ii) At what points (x, y) in the plane are the function $f(x, y) = \sin \frac{1}{xy}$ is continuous.