



K24U 2752

Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/
Improvement) Examination, November 2024

(2019 to 2022 Admissions)

CORE COURSE IN MATHEMATICS

5B07MAT : Abstract Algebra

Time : 3 Hours

Max. Marks : 48

PART – A

Answer any 4 questions from this Part. Each question carries 1 mark. (4×1=4)

1. Find the number of elements in the cyclic subgroup of \mathbb{Z}_{30} generated by 25.
2. Give an example of an infinite group that is not cyclic.
3. Define alternating group.
4. Let $\phi : S_n \rightarrow \mathbb{Z}_2$ defined by $\phi(\sigma) = \begin{cases} 0, & \text{if } \sigma \text{ is an even permutation} \\ 1, & \text{if } \sigma \text{ is an odd permutation} \end{cases}$
Compute $\ker \phi$.
5. Describe all units in the ring \mathbb{Q} .

PART – B

Answer any 8 questions from this Part. Each question carries 2 marks. (8×2=16)

6. Prove that identity element in a group G is unique.
7. Describe all the elements in the cyclic subgroup of $GL(2, \mathbb{R})$ generated by $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
8. Find the number of generators of a cyclic group having order 60.

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9. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ be a permutation in S_6 . Compute $\langle \sigma \rangle$.
10. Find all orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$ in S_6 .
11. Prove that every group of prime order is cyclic.
12. Let ϕ be a homomorphism of a group G into a group G' . Then prove that
 i) if e is the identity element in G , then $\phi(e)$ is the identity element e' in G' .
 ii) if $a \in G$, then $\phi(a^{-1}) = (\phi(a))^{-1}$.
13. Let G be a group, and let $g \in G$. Let $\phi_g : G \rightarrow G$ be defined by $\phi_g(x) = gxg^{-1}$ for $x \in G$. Prove that ϕ_g is a homomorphism.
14. State the fundamental homomorphism theorem.
15. Solve the equation $x^2 - 5x + 6 = 0$ in \mathbb{Z}_{12} .
16. Is every integral domain a field? Justify.

PART - C

Answer any 4 questions from this Part. Each question carries 4 marks each. (4×4=16)

17. Prove that a subset H of a group G is a subgroup of G if and only if
 i) H is closed under the binary operation of G .
 ii) the identity element e of G is in H .
 iii) for all $a \in H$ it is true that $a^{-1} \in H$ also.
18. State and prove division algorithm for \mathbb{Z} .
19. Find all subgroups of \mathbb{Z}_{18} and draw the subgroup diagram for the subgroups.
20. Prove that every group is isomorphic to a group of permutations.
21. Let H be a subgroup of G . Let the relation \sim_L be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Prove that \sim_L is an equivalence relation on G .



22. Show that an intersection of normal subgroups of a group G is again a normal subgroup of G .
23. Define subring. Let R be a ring and let a be a fixed element of R . Let $I_a = \{x \in R/ax = 0\}$. Show that I_a is a subring of R .

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. (2×6=12)

24. a) Let G be a cyclic group with n elements generated by a . Let $b \in G$ and let $b = a^s$. Prove that
- i) b generates a cyclic subgroup H of G containing n/d elements, where d is the greatest common divisor of n and s .
 - ii) $\langle a^s \rangle = \langle a^t \rangle$ if and only if $\gcd(s, n) = \gcd(t, n)$.
- b) Find all generators of \mathbb{Z}_{12} .
25. a) List the elements in the dihedral group D_4 .
- b) Find all subgroups of D_4 of order 2.
26. a) Let H be a normal subgroup of G . Prove that the cosets of H form a group G/H under the binary operation $(aH)(bH) = (ab)H$.
- b) Find the order of $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.
27. Prove that $F = \{a + b\sqrt{2}/a, b \in \mathbb{Q}\}$ with usual addition and multiplication forms a field.
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